Submarine Propulsion Shaft Life: Probabilistic Prediction and Extension through Prevention of Water Ingress

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Submarine propulsion shafts have demonstrated acceptable reliability performance when inspected and refurbished at least every 6 years. Designers wish to extend the inspection interval to 12 years without sacrificing reliability. This interval is unprecedented, as no known submarine shafting system is currently operated with this inspection cycle, nor are any known commercial vessel shafts. Experience and improved design have eliminated many threats to the life of a submarine shaft, but inspections of existing shafts show a high percentage with signs of wetting, leaving designers with less-than-acceptable confidence to approve this longer inspection interval due to the possibility of corrosion fatigue failure. This thesis uses probabilistic models from literature for pitting and cracking of wetted shafts, along with Monte Carlo simulations, to predict results of shafts inspections. Each possible water ingress distribution is analyzed by simulating shafts under 6 years of exposure to the water ingress, pitting, and cracking models in order to estimate the effects of corrosion fatigue. A water ingress distribution that predicts inspection results closest to actual inspection results is identified. Some information about water ingress is inferred from this distribution. Next, using the same literature models, a water ingress distribution that predicts similar performance at 12 years is identified. It is shown that the time a shaft is in service prior to becoming wetted must increase substantially. Predicted failure rates are low, but they are still higher than acceptable. This thesis recommends that inspection procedures are updated to provide more robust information for future analyses, which would better identify the appropriate distributions and greatly reduce uncertainty.

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Submitted to the Departments of Mechanical Engineering and Materials Science and Engineering in Partial Fulfillment of the Requirements for the Degrees of Naval Engineer's Degree in Naval Construction and Engineering and Master Of Science in Materials Science and Engineering

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Symbol	Name	Units
a	Pit size (depth)	m
a_0	Initial pit depth	m
c	Characteristic pit or crack size	m
ΔH	Activation enthalpy	kJ
ΔK	Stress intensity factor (range)	$MPa/m^{1/2}$
Δσ	Stress (range)	MPa
f	Frequency	Hz
F	Faraday constant	J/(v g)
Hz	Measure of frequency	Cycles/sec (1/s)
I_{P0}	Initial pitting current	mA/cm^2
k	Number of particles	
M	Molecular weight	g/mol
n	Valence	
N	Number of cycles	
φ_k	Aspect ratio	
r	Pit size (radius)	
ρ	Density	Kg/m^3
R	Universal gas constant	J/(mol K)
t_{pg}	Time for pit growth	days

1.0 Introduction

Of critical interest to many industries is the reliability of components and systems. Reliability and service life prediction are inherently cross-disciplinary and complex topics. In general, engineers are able to design in adequate margins to deter known and anticipated failure modes. However, failures continue to occur that necessitate further changes to designs and systems. All too often,, unanticipated failure mechanisms are discovered after parts and machines are in service, and reliability analysis tends to be a business of hindsight and lessons learned.

The world of ships and submarines, including those of the military, is not immune to the occurrence of failures. The submarines of many countries rely on a single propulsion shaft, making this shaft vital to the missions and effectiveness of these vessels. Moreover, a shaft failure that allows gravity or drag to unseat the broken shaft and remove it from the vessel creates a large diameter flooding penetration that is effectively impossible to plug, ensuring destruction of the submarine, and in a timeframe likely to claim the lives of all aboard, even if the vessel had been operating on the surface.

In spite of the best efforts of designers, there have been a number of submarine shaft failures. Designs have been continuously improved, and recent classes all but eliminate the possibility of shaft ejection, even if it fractures. The number of historical failures is a statement about the complexity of the design and operations of these components: multiple modes of failure exist simultaneously, creating a very constrained design space. These mechanisms of failure are most often the result of complex interactions between geometry, materials, environment, loading, and many other factors, and therefore anticipating and quantifying their effect on systems is difficult. Establishing service life is often based on information and data from sources other than physical operation of the shaft in the ocean environment. Fitness for service analysis therefore requires extrapolating from a non-operating (laboratory data, simplified experiments, etc.) domain where experimental data is available or can be taken, to an application domain where there is little to no data. Often, obtaining application level data is prohibitively difficult or expensive (King, Arsenlis, Tong, & Oberkampf, 2012). In the case of submarine propulsion shafting, the size of the components, the length of time in service of the systems, and the highly variable operational environment all complicate, or outright prevent, direct testing. Scaled and simplified tests are performed instead, in an attempt to gain understanding of the issues, and the results are extrapolated to operating conditions. This is especially true in the areas of corrosion effects and corrosion testing.

Test results are very often used to develop models, which in the case of corrosion testing must be calibrated against extensive corrosion data, either with known environmental conditions or in situations where designers are capable of having these conditions established retroactively. (Melchers, Probabilistic Models for Corrosion in Structural Reliability Assessment - Part 2: Models Based on Mechanics, 2003). Such data is not always readily available, and the business of extrapolation is more of a physics endeavor than a statistics endeavor, requiring in some ways even more depth of understanding of the processes involved (King, Arsenlis, Tong, & Oberkampf, 2012). Another method

might be to integrate existing data from other sources, but that also is not without its pitfalls. A review of attempts to pool models for corrosion indicated that such pooling produces poor-quality models exhibiting large amounts of scatter. (Melchers, Probabilistic Models for Corrosion in Structural Reliability Assessment - Part 2: Models Based on Mechanics, 2003). Further, extrapolation methods do not deal with missing physics, and it is a common experience that solving one problem only reveals previously unknown couplings, physics, or failure modes (King, Arsenlis, Tong, & Oberkampf, 2012). This has been witnessed in submarine shaft maintenance and design, where repeated solutions have failed to provide the full expected increase to service life, as new failure modes – previously masked by modes with shorter time scales – come to control the service life and failure rate.

1.1 Existing Limit on Shaft Life

Past shaft failures on submarines have been systematically evaluated and their causes addressed throughout the history of the submarine service. Several reviews, often at design decision points for new classes or improvements to existing submarines, have been performed, each isolating the primary mechanism or mechanisms responsible for failures that limit the service life of the shaft. However, solutions to these limiting phenomena have served often to expose additional underlying mechanisms and failure modes – new physics as predicted by King et al., above. Notwithstanding this history, the navy currently has a class of submarines with 30 years of operating experience with no shaft failures. There have, however, been many cases where a shaft has developed precursors to failure by corrosion fatigue (pits, small cracks, etc.). Shafts are removed and inspected during scheduled maintenance periods in a drydock, with a 6-year maximum operational time on a shaft. At the end of each operational period, the shaft is removed from service for refurbishment and then returned to the rotating stock of propulsion shafts. Refurbishment consists of removal of all protective coatings and wear sleeves, followed by inspection through non-destructive testing and repair of all unacceptable conditions (defects and indications) identified. This 6-year limit is driven by concerns about corrosion fatigue, a process initiated by water gaining access to the carbon steel of the shaft to cause corrosion. Inspections of these shafts, though free of failures, confirm that corrosion fatigue does progress and needs to be monitored, particularly in the regions of concern indicated in Figure 1, which is a simplified schematic of the current propulsion shafting arrangement. In this figure, the shaft configuration aft of the dry, pressurized engine room is illustrated. The shaft passes through two bearings, each with an alloy 625 (an Inconel) sleeve. These sleeves exhibit exceptional corrosion performance, and are used as the wear surfaces in contact with the lubricated bearings. The stern tube bearing, on the right in the figure, is the transition point from the dry engine room to the wet ballast and mud tanks. Aft of this bearing (towards the left in the drawing), all spaces in the illustration are free-flood spaces, exposed to sea water at submergence pressure. The propeller, in the far left of the figure, is attached to the shaft. From the propeller bearing aft, the remaining length of shaft and propeller are suspended with no further supports, creating a strong bending force, often modeled as a cantilevered beam. Each revolution of the shaft for propulsion, however, changes the orientation of this bending moment,

relative to any point on the surface of the shaft, through a full cycle of bending (from maximum tension to maximum compression and back). Though the shaft is almost completely encased in a glass-reinforced plastic (GRP) coating, the figure indicates that at each waterborne end of each of the sleeves, water sometimes gains access to the shaft steel, and combines with this cyclic bending load (as well as the torsional load of propulsive forces) to create the conditions that lead to corrosion fatigue.

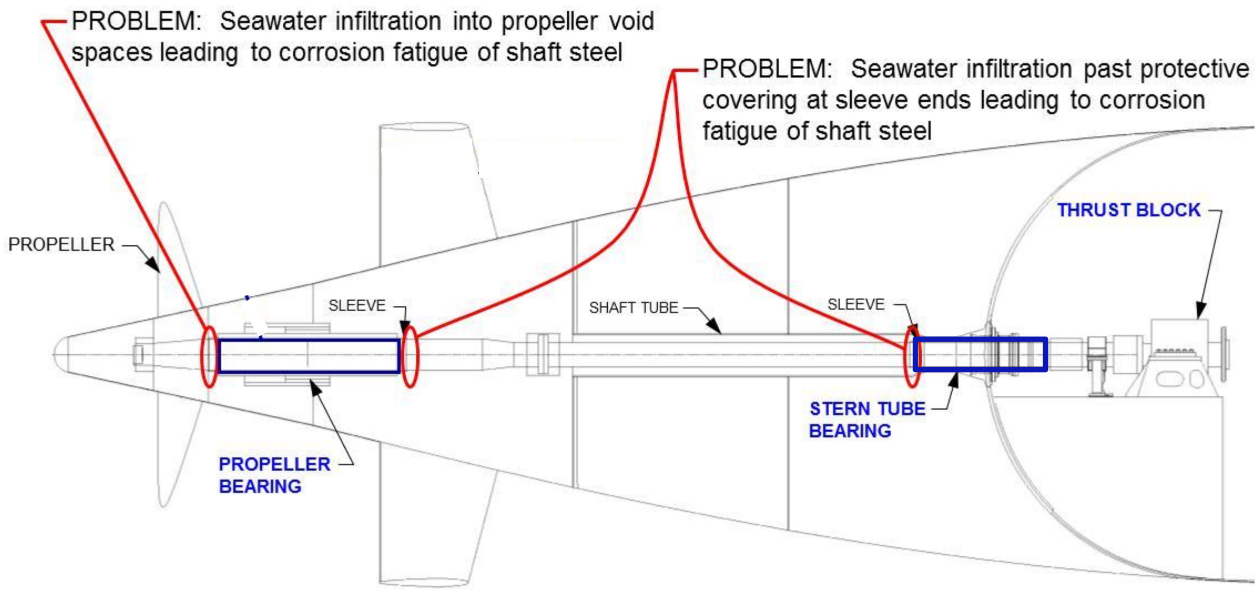

Figure 1: Schematic of submarine shafting indicating regions of corrosion fatigue concerns[1]

The submarine community has elected to increase the propulsion shaft inspection interval for the next class of submarines, now in the beginning stages of design, to have scheduled availability in a drydock every 12 years, instead of 6. This requires a substantial increase in the service life of the propulsion shaft, but it is a key to saving billions of dollars in the procurement, operations, and maintenance of the vessels. To achieve this goal, the navy needs to better understand and design against corrosion fatigue of these components. One researcher gives a summary which captures the difficulty of this task:

> *"In practice corrosion is not an independent issue. Corrosion interacts with applied stresses, fatigue, mechanical damage, and most importantly, with protective systems such as cathodic protection, paint coatings, and management practices. The interaction with each of these phenomena or materials is generally complex and the interactions are not fully understood in most cases. There is considerable scope for further fundamental and applied corrosion research. Eventually this will need to be*

[1] Taken from "Shaft Life Advancements", W. H. Needham, Presentation at Shaft Life Advancement Industry Day at MIT, October 13, 2011.

> *translated into engineering design rules and guides for the "protection" of ageing infrastructure, including the development of probabilistic models.*[2]

The submarine community finds itself looking for precisely this kind of probabilistic model to evaluate design options and to explore sources of uncertainty that can be reduced that will help achieve its aggressive shaft service life goal. Shi and Mahadevan (2001) identify three methods to ensure component reliability: a "safe life" method requiring the structure to survive under a given loading for a specific number of service cycles, essentially a mechanics only condition; the "fail safe" approach that requires the entire structure be capable of damage without catastrophic failure of the entire structure; and a "damage tolerance" approach assuming an initial flaw or defect that grows, but the growth of which is not adequate to endanger the structure during the design or service life and can be found by inspection and repaired. The third approach is most applicable here, with corrosion damage and pit formation filling the role as the initial damage, with the potential of transitioning into cracks that must not be allowed to grow until they endanger the shaft.

An example of this process is found in the reliability analysis of steam generators in nuclear power plants. Analysis of steam generator tube failure data reveals that failures were derived from multiple sources, including stress corrosion cracking, fretting, and damage from foreign objects. The most prevalent source of failures, however, was cracks in the roll transition region of the tube sheet-a situation analogous to the shaft degradation process and the challenge of the submarine force, primarily concerned with corrosion leading to cracking (Pitner, 1988). In the case of these nuclear steam generators, there is a large and expanding database from inspections that allows for ever-improving statistical and engineering analysis. Instead of a large database of failure history, shaft inspection data is available from only approximately 60 shaft inspections. Unfortunately, the quantity and quality of data available are both considerably less than Pitner was able to obtain. A single, comprehensive analysis of data is not an accessible solution, so the problem of corrosion pits and inspection intervals to preclude failures from corrosion pitting and fatigue cracking must be tackled through other methods.

1.2 Complexity of the Problem

The problem of corrosion fatigue failure that faces the submarine community is not uncommon: after conducting a thorough review across many applications, corrosion pitting was found by one group of researchers to be responsible for nucleating fatigue cracks in a wide range of metals (Chen, Wan, Gao, Wei, & Flournoy, 1996). Expanding on both the ubiquity and complexity of the problem, another pair of researchers declared that fatigue crack initiation and growth had been found to degrade reliability of many structures subjected to repeated loadings. They further state that the data on this process exhibits considerable scatter, creating a significant challenge for the design for reliability, which needs to

[2] Melchers, "Probabilistic Model for Marine Corrosion of Steel for Structural Reliability Assessment," 2003. P 1492.

recognize appropriate extreme value behavior (3-sigma reliability or other metrics giving a small probability of failure) (Tryon & Cruse, A Reliability-Based Model to Predict Scatter in Fatigue Crack Nucleation Life, 1998).

Corrosion fatigue requires a series of events, sometimes referred to as a failure chain or event tree, to proceed in succession. Each step involves different physics and is controlled by different parameters and interactions of the many variables involved. Figure 2 depicts the corrosion fatigue sequence of events that limits the submarine shaft service life, and lists a few of the challenges that complicate each step. Though the shaft system has a number of protective systems and features in the design, much of the system is submerged in seawater, as shown in Figure 1, and water eventually reaches the mild steel of many shafts, beginning the corrosion fatigue process. The mild shaft steel, when exposed to this seawater environment, corrodes, and that sometimes leads to the formation of pits. These pits act as stress concentrators for the various loads on the shaft, and sometimes cracks form, then propagate, leading to one failure mechanism.

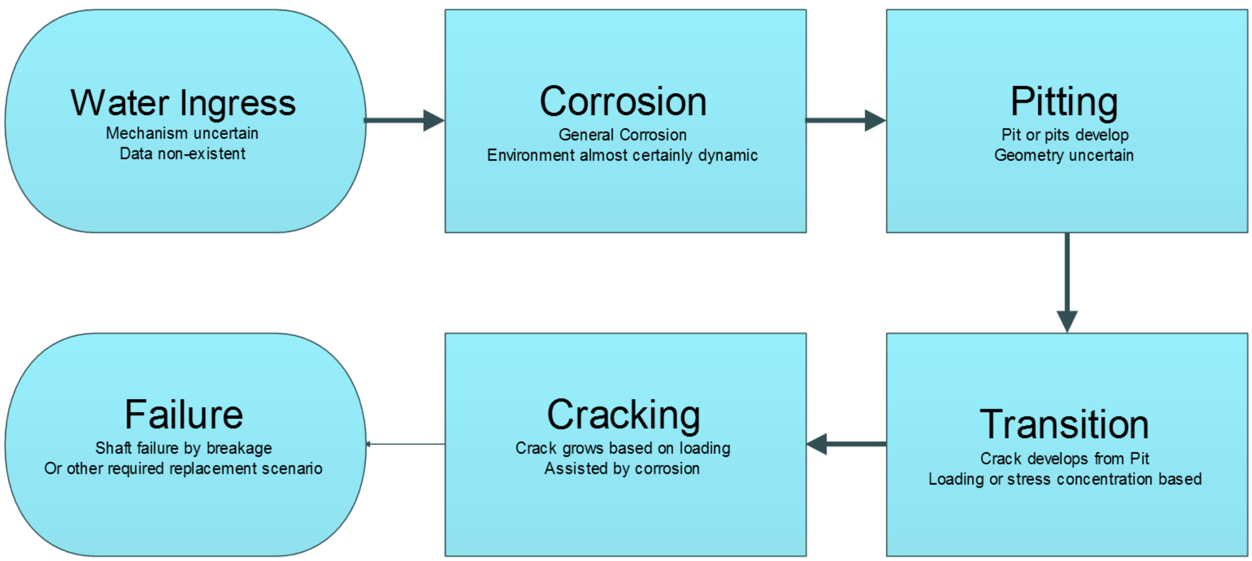

Figure 2: Corrosion fatigue process

The focus of this thesis is the development of a model that provides information to help estimate the required inspection interval for shafting. Water ingress, and its timing, is critical to the analysis of shaft life. By modeling each of the subsequent steps, possible distributions of water ingress may be analyzed. Due to the many combinations of materials, environments, environmental factors, and types of corrosion, only limited data is usually available for a given material exposed to a particular environment. This is the case for mild steel under marine conditions (Dechema, 1992) including submarine propulsion shafting. Melchers (2003), a structural engineer, states that much of the data that is available comes from short-term tests under laboratory conditions. He goes on to state that, though the literature on corrosion is extensive, conventional corrosion theory consists mainly of general principles and

electrochemistry and is applicable mainly to short-term corrosion tests under specific and often ideal conditions. Unfortunately, such data is seldom able to provide practical information relevant for use by structural engineers, such as the amount of material likely to be lost for particular structural details under particular exposure conditions (Melchers, Probabilistic Model for Marine Corrosion of Steel for Structural Reliability Assessment, 2003). A common practical approach is thus to consult compendia based on experience (Dechema, 1992) or to conduct coupon exposure tests in a specific environment, the results of which are then used to project likely future corrosion behavior. Melchers closes his critical analysis by stating that both methods can lead to corrosion rates that are not accurate for the timeframes to which they are applied (Melchers, Probabilistic Model for Marine Corrosion of Steel for Structural Reliability Assessment, 2003).

Examining the areas of concern for corrosion pitting, there are additional complexities with which to contend. Figure 3 provides a more detailed view of one of these areas. This figure illustrates the aft portion of the shaft as it exits the stern tube bearing. There is an alloy 625 (Inconel) sleeve, used as a wear surface for the bearing interaction, as seen in the drawing. This bearing is placed on the shaft using a shrink-fitting technique, and then a GRP protective layer is applied covering the sleeve-shaft interface and the length of the shaft. The area labeled as a typical corrosion area indicates where inspections have revealed many defects, typically referred to only as “indications” on an inspection report. The path the water takes to access this area is not yet known. It is also not known if there is free exchange of fresh seawater into the area once penetrated, or if the water stagnates in the small geometry created. It is therefore unknown if this region under attack is an aerobic environment, an anaerobic environment, or possibly one in which the intitally available oxygen becomes depleted, each of which would have a different corrosion response. Other work by this project has revealed that water in this region may complete a galvanic circuit between the sleeve material and the shaft steel, which would indicate a very high corrosion rate that might be brief or might endure long enough to create significant damage. Each stage of the event tree in Figure 2 has similar complications, making the modeling of the corrosion fatigue process in this case difficult.

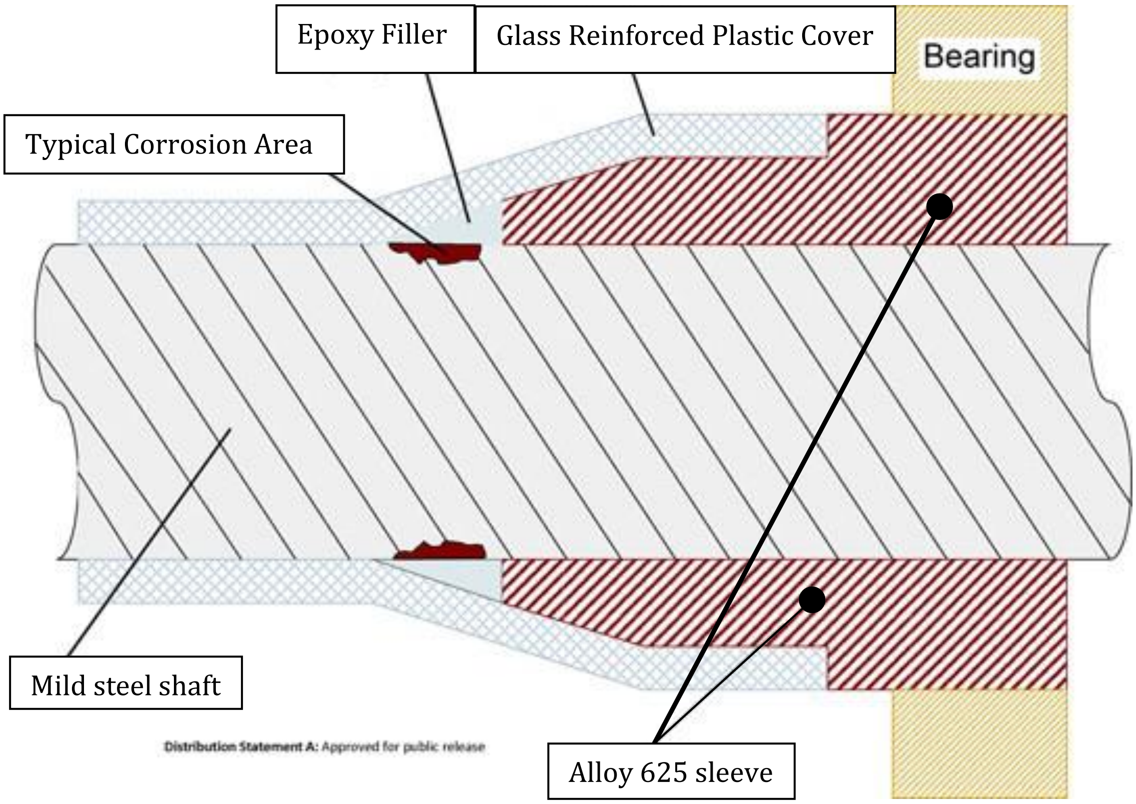

Figure 3: Detail of shaft/sleeve interface, highlighting region of concern[3]

1.3 Possible Solutions

Due to the above considerations, the task faced by submarine designers to mitigate environmental degradation of the shaft material is difficult. There are a number of possible solutions, each of which might partially or completely achieve success. For example, increasing the detail of the inspections to provide more robust information would give designers a stronger footing from which to predict performance of the existing shaft system. For example, it is not currently known if a particular "indication" is a pit, a pit with a crack, a machining artifact, or another of several possibilities. The ability to characterize the distributions of indications may allow designers to develop more robust life prediction models to evaluate the likely time to failure for the existing system. As will be discussed, however, the limited data available gives little promise that this method alone will provide confidence in the current shaft design with a 12-year shaft inspection interval.

[3] Taken from "Maintenance Free Technologies Overview", Dr. Airan Perez and Edward Lemieux, Presentation at Shaft Life Advancement Industry Day at MIT, October 13, 2011.

According to the information provided by the submarine community, the current design has accounted for and effectively eliminated all purely mechanical sources of failure known to have previously affected propulsion shafts. If the shaft can be kept dry with high confidence, therefore, the longer shaft life will likely be achieved. Designers, unfortunately, have little information regarding the current time or mechanisms of water ingress, and essentially no existing data on effectiveness of current or proposed systems to prevent water from accessing the shaft metal. However, preventing water ingress is an attractive solution for achieving a longer service life, as it requires few significant changes to the design of the shaft itself, and interrupts the failure chain depicted in Figure 2 at the earliest possible point.

There are other solutions; the shaft design itself could be changed in ways that interrupt the failure chain elsewhere. Incorporating materials that are less susceptible to corrosion, or perhaps immune to pitting in the operational environment, would reduce or eliminate the likelihood of corrosion fatigue failures. Research on pipelines shows that, after the transition from pits to cracks has occurred in the field, tiny, elongated, blunt cracks are often seen in very large numbers and frequently in crack colonies. The majority of these cracks become dormant, but if they surpass a threshold depth, around 0.5mm, they can propagate and may lead to pipeline rupture if not detected and removed. (Fang, Eadie, Elboujdaini, & Chen, 2009). It might be possible to design a shaft that causes even more cracks to become dormant, or in which the threshold is higher. As the scale of design changes grows, however, a conflict quickly arises between making changes believed to solve the current problem and the added risks of new problems being exposed, alluded to by King et al. (2012) and previous shaft life experiences.

Submarine designers have revealed that, in order to progress through the procurement process on schedule, there is an immediate need to establish confidence in the ability to achieve a 12-year maintenance cycle. For this reason, solutions requiring less expansive testing and validation are preferred over solutions requiring longer programs of study and analysis. Major design changes, and truly exotic solutions such as shaftless propulsion, are therefore beyond the scope of this project, although their long term pursuit is recognized as having value for subsequent classes of submarines, where the design and testing windows might better facilitate them. To that end, this project has also performed a limited investigation into the feasibility of developing a cladding material that would largely preclude pitting, and which could be evaluated and tested in a time frame for the future submarine classes. However, the focus of the project, and this thesis, is on the immediate needs of the class currently being designed.

This thesis infers information about water ingress for the existing design by coupling models for subsequent steps of the failure chain with summary data from the shaft inspections performed to date. It then makes a first order prediction of the failure distribution for the existing shaft design, if they were to be left in service without refurbishment. Finally, the same models will be used to evaluate the required water ingress distribution that must be achieved, assuming no other major changes in the shaft design, such that a 12-year service life yields similar predicted inspection performance.

2.0 Existing Models and Life Predictions

To develop the models needed for this study, literature on the subject and on each phase in the failure chain is considered. The information available to engineers about marine corrosion, for example, is largely anecdotal, not well organized, and of limited use even for simple applications, according to Melchers. Although classification societies and a number of navies regularly collect plate thickness measurements as estimates for corrosion loss, little of the data has been published for a variety of reasons (Melchers, Probabilistic Model for Marine Corrosion of Steel for Structural Reliability Assessment, 2003), though a review of published corrosion statistics for ships is available (Melchers, Probabilistic Models of Corrosion for Reliability Assessment and Maintenance Planning, 2001). More specific to the immediate concern of corrosion fatigue, field evidence suggests that corrosion pits might be a common site for crack initiation. In one laboratory study, the earliest cracks appeared to initiate at corrosion pits forming around non-metallic inclusions; later cracks grew from corrosion pits that formed randomly on the surface (Fang, Eadie, Elboujdaini, & Chen, 2009). The models of these researchers and others are considered in this section.

A distinction must be drawn between corrosion pits, of concern here, and pitting corrosion. In mild steels in a corrosive environment, anodic and cathodic areas tend to move around on the surface to create the impression of uniform corrosion often referred to as "general corrosion" (Melchers, Probabilistic Models for Corrosion in Structural Reliability Assessment - Part 2: Models Based on Mechanics, 2003). However, the level of uniformity is subjective, and various localized surface geometries may develop. In stainless steels, aluminum alloys, and several other corrosion-resistant metals, this general corrosion is significantly resisted by the formation of passive, protective layers, often oxides. In locations where the protective layer is breached, corrosion may be rapid and highly localized, burrowing deeply into the metal, creating a pit with a very high depth-to-diameter aspect ratio. This is called pitting corrosion, and is not of primary interest here, as the shaft is a mild steel. Even for the general corrosion of mild steel, buildup of a complex corrosion product film on the surface of the corroding metal will soon control the behavior by inhibiting the supply of oxygen to the corrosion interface even for fully aerated waters (Melchers, Probabilistic Models for Corrosion in Structural Reliability Assessment - Part 2: Models Based on Mechanics, 2003). In this environment, local areas with higher corrosion rates may develop, creating depressions in the surface that often take the form of shallow, low-aspect ratio pits, called corrosion pits. It should also be noted that galvanic couples between the shaft steel material and more noble metals can cause highly localized corrosion. In fact, as indicated in Figure 3, the region of the shaft immediately adjacent to the alloy 625 bearing sleeve is one such area, and such a couple is suspected based on other work by this project, though that work is not detailed in this thesis. Under load, especially a cyclic load, these corrosion pits may affect the stress concentration and response of localized regions, including the formation of cracks. This is the pitting that is of concern in the current research, as one of the steps in the corrosion fatigue failure chain depicted in Figure 2.

In the remainder of this chapter, several views of the entire process of corrosion fatigue will be discussed, followed by a more detailed review of existing treatments in the literature for each step in the failure chain. Finally, as it will be shown to be of deep concern, a general treatment of uncertainty as it relates to the development of models and to predictions from those models will be evaluated.

2.1 Selection of a Framework

Fatigue cracks are very often observed to nucleate and propagate from corrosion pits (Shi & Mahadevan, 2001). Many researchers have studied this important phenomenon, with varying methods and resulting conclusions. One paper concluded that, “in the field, it generally takes years for pits to grow and initiate cracks, and the pit growth may proceed under intermittent exposure conditions” (Fang, Eadie, Elboujdaini, & Chen, 2009). Another group contended that there is a competition between time spent in pit growth and crack growth, citing the results in Figure 4, which show that longer times spent growing (larger) pits correspond to greatly reduced growth times for the cracks that initiate from these pits:

The effect of pit size on the corrosion fatigue life

Transition size from pitting to crack nucleation c_{ci} (mm)	Pit growth time (days)	Short crack growth time (days)
0.05	69	6690
0.1	554	3860

Figure 4: Demonstration of effect of larger pits on crack growth duration[4]

In Kondo (1989), who is very often referenced as a starting point for other models, the author assumes that failure occurs in three stages: pit initiation and growth, crack initiation from the pit, and crack propagation. In another example, researchers first performed a then-exhaustive review of models and solutions (Shi & Mahadevan, 2001). This pair then built on work from several authors: a three-stage model from one source (Harlow & Wei, Probability Approach for Corrosion and Corrosion Fatigue Life, 1994), a seven-stage model proposed but not numerically developed (Goswami & Hoeppner, 1995), Harlow and Wei’s probabilistic pit corrosion model (reviewed in several sections of this thesis), a development of Kondo’s transition model by Chen et al. (also detailed in this thesis), and a series of other studies. Shi and Mahadevan, who define both short and long crack stages, conclude that short crack growth rates exceed those of long cracks – thereby necessitating the separation of the two in their model (Shi & Mahadevan, 2001).

[4] Taken from Shi and Mahadevan, “Damage Tolerance Approach for Probabilistic Pitting Corrosion Fatigue Life Prediction,” 2001, p. 1499.

Some of the most comprehensive work is done by Australian Robert Melchers, who informs his readers that future models must be probabilistic, to account for uncertainties caused by: modelling approximations; variability in environmental conditions and in modeling them; and variations in material (Melchers, Probabilistic Models for Corrosion in Structural Reliability Assessment - Part 2: Models Based on Mechanics, 2003). In the analysis of this thesis, the goal is to make use of the best probabilistic models, heeding Melchers's instruction. Melchers goes on to state that variability is due to a number of sources, but unfortunately there are very few suitable data available, going on to say that even for variability between coupons at the same site, most published reports give insufficient information for its estimation, typically reporting the mean of (usually only) two coupons and not even the individual results (Melchers, Probabilistic Model for Marine Corrosion of Steel for Structural Reliability Assessment, 2003). Evaluation and selection of models for this paper, then, must consider treatment of variability, as well.

Returning to the summary work of Shi and Mahadevan, they conclude that the fatigue life of a component in a system is the sum of four critical phases: time to pit nucleation, time for pit growth leading into short crack nucleation, time for short crack growth, and time for long crack growth. Their model also includes transitions between these times as additional stages, as illustrated in Figure 5 (Shi & Mahadevan, 2001).

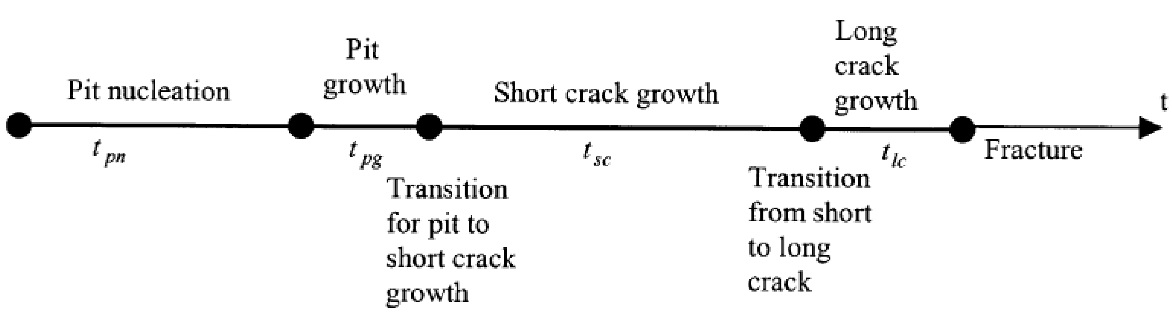

Figure 5: One model considering 7 stages, four discrete phases in time and three transitions[5]

Evaluation of the sometimes anecdotal information from the submarine shaft inspections reveals that few cracks have developed, none of which have propagated to failure. While good news from the standpoint of reliability, this also means that very little information is available for the calibration and/or validation of detailed crack modeling results. For this reason, detailed consideration focuses on the following phases, consistent with the chain presented earlier: corrosion, primarily as it becomes a source of uncertainty; pitting, both nucleation and growth; and transition from pits to cracks. A

[5] This figure is taken from Shi & Mahadevan, "Damage tolerance approach for probabilistic pitting corrosion fatigue life prediction," pp 1495.

simplified crack growth model is used for first order failure predictions once water ingress distributions are identified for both 6 and 12 year service lives.

2.2 Corrosion and Corrosion Rate

Melchers's review of published corrosion loss data for structural steel coupons in immersion conditions immediately reveals that corrosion is not linear in time, and shows very large scatter. He concludes that "corrosion rate" has limited meaning and that a rate measured over a short time may be quite misleading in predicting longer-term corrosion. It also follows (due to the observed scatter) that any probabilistic models based on such data will have a high level of uncertainty and be of limited use (Melchers, Probabilistic Model for Marine Corrosion of Steel for Structural Reliability Assessment, 2003). In later work, Melchers proposes a more complex model for corrosion based on review of many studies. The general model is schematically depicted in Figure 6:

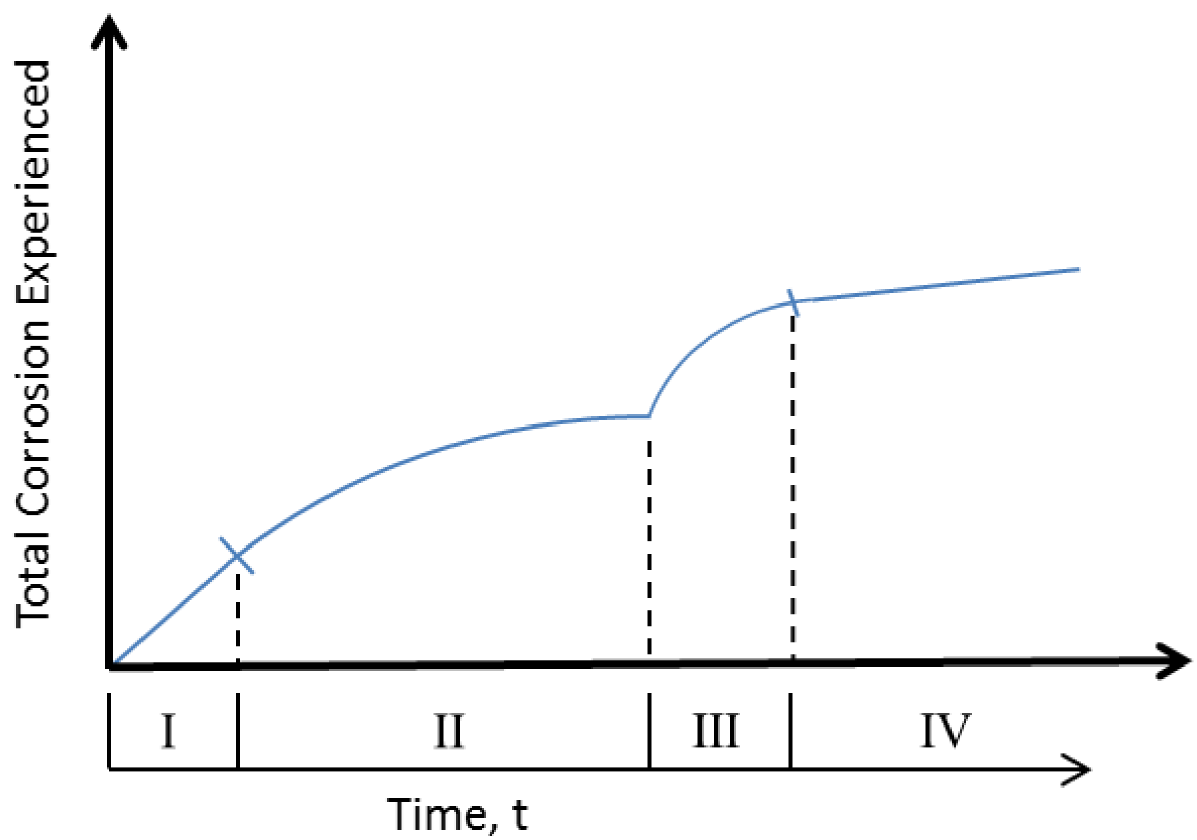

Figure 6: Melchers's model for corrosion over extended periods of time[6]

A detailed description of the model, paraphrasing the author's longer explanation, follows. Phase 1 in his model is called the kinetic phase, and consists of the time immediately following immersion. Initially, a rapid increase in corrosion rate (from zero) quickly leads to a steady rate of corrosion, which is indicated by the slope of the observed linear region. Usually under oxygen concentration control, this rate is the value commonly referred to and tested as the "corrosion rate." Phase 2 develops as a buildup of film (corrosion products) limits the diffusion of oxygen to the base metal, such as diffusion control through rust in the mild steel case. Phase 3 is when biological organisms and other organic processes, especially sulfate reducing bacteria (SRB), take over to again increase the corrosion rate. He notes that no models exist for this non-linear region that is dependent on numerous parameters. Phase 4 is the asymptotic, long term corrosion behavior. Commonly, Phases 1 and 2 are of the greatest practical interest. However, he also notes that Phases 3 and 4 may be of primary interest in tropical waters. (Melchers, Probabilistic Model for Marine Corrosion of Steel for Structural Reliability Assessment, 2003). Developing this model, he does note that SRB regions are likely to be under activation control, as these bacteria operate independent of oxygen; hence the long term rates are often dependent on metal composition and temperature more than other factors (Melchers, Probabilistic Models for Corrosion in Structural Reliability Assessment - Part 2: Models Based on Mechanics, 2003).

Acknowledging these complications, the analysis in this thesis uses published rates and statistics on variability, but recommends side-by-side experiments using natural and artificial seawater environments for future work.

2.3 Pit Nucleation and Growth

Much of the research that deals with pitting in detail is concerned exclusively with pitting corrosion (discussed/defined in the beginning of this chapter), and is therefore of limited applicability to the mild steel of submarine shafts. Additionally, pit nucleation distributions are often simply assumed or treated deterministically. Kondo, for example, "develops" pits according to the deterministic model that the radius of observed pits is given by: $r = t^{1/3}$. This model, then, implies that a virgin surface nucleates minute pits as soon as it goes into service (Kondo, 1989).

A probabilistic method was used by Shi and Mahadevan. In their model, consisting of seven stages, they stated that time to pit nucleation depends on numerous factors which were not yet well understood. They therefore treated the time to pit nucleation and the size of initial pits as random variables, and

[6] Although he references this model in many of his works both before and afterwards, this depiction was taken from his second 2003 paper, partially titled "Part 2: Models Based on Mechanics," p 273.

then tested several possible distributions of each. By comparing the results with these various nucleation distributions to field experiences, they were able to infer which distributions might be likely (Shi & Mahadevan, 2001). As this method is similar to the water ingress method used in this thesis, the analysis of this thesis uses the distributions that the authors identified as most consistent with experimental data for pit nucleation, rather than trying to distinguish from among the effects of several simultaneously changing distributions.

The question of geometry is central to many discussions on pits and pit growth. Almost all authors assume a somewhat idealized geometry. Kondo (1989), for example, assumes hemispherical pits. Harlow and Wei (1998) derive their growth formula assuming ellipsoidal pits, and they take three approaches to handling aspect ratio as each pit grows. Their first method is to assume a fixed aspect ratio, from which they derive the following pit growth model, equivalent to those of several other authors:

$$\frac{2}{3}\pi \emptyset_k^2 (a^3 - a_0^3) = \frac{M I_{p_0}(k)}{nF\rho} exp\left[-\frac{\Delta H}{RT}\right] t \tag{1}$$

where k is the number of constituent particles initiating a given pit, $\emptyset_k$ is the aspect ratio, a is pit depth, a_0 is initial pit depth, M is molecular weight, $I_{p0}(k)$ is the initial pitting current (a function of k), n is the valence, F is Faraday's constant, ρ is the density, ΔH is the activation enthalpy, R is the universal gas constant, T is temperature in kelvin, and t is time. The other two treatments of aspect ratio provide complex solutions and are not considered in detail in their analysis, so they are omitted here (Harlow & Wei, A Probability Model for the Growth of Corrosion Pits in Aluminum Alloys Induced by Constituent Particles, 1998).

Equation 1 appears in several papers reviewed for this research, though this is the most general form. In this formulation, taking $\emptyset_k = 1$ yields the hemispherical assumption, which is used often by other authors. In a study using an accelerated method to generate pits, it was clear after inspection that the pits generated were nearly circular on the surface, and semi-circular in cross-section, giving support to the simplest geometry (Fang, Eadie, Elboujdaini, & Chen, 2009). After some work, even Harlow and Wei assume hemispherical pits, but note that their sample of more than 1500 pits gave an average aspect ratio, $\emptyset_k$, of 1.57, with a range of 1.0 to 4.2, so they intended to consider ellipsoidal pits in future work (Harlow & Wei, A Probability Model for the Growth of Corrosion Pits in Aluminum Alloys Induced by Constituent Particles, 1998). More complex geometries might be justified in the future if further details become available from better shaft inspection data, but as stated earlier, only somewhat vague counts of "indications" are available for the analysis in this work. For this reason, and due to its ubiquity and acceptance for first-order evaluations, hemispherical pits are assumed in this analysis. Additionally, a strong argument can be made for treating $\emptyset_k$ itself as a random variable, but pragmatism leaves it being treated deterministically in almost all published modeling (Harlow & Wei, A Probability Model for the Growth of Corrosion Pits in Aluminum Alloys Induced by Constituent Particles, 1998), as it will be treated here.

There are few pit growth models in the literature that differ substantially in form from the model developed by Harlow and Wei (1998). These two authors begin with a probabilistic distribution of constituent particles based on scanning electron microscope images of titanium. Their pit growth model has a probabilistic initial current dependent on the clusters of these particles, modeled as a Pareto distribution (see Appendix A for discussion of this distribution). Referring to pits as initial damage, they assume this damage nucleates on the bare surface as a pit due to a localized galvanic corrosion cell surrounding exposed constituent particles in the alloy. Their work includes an argument that only cathodic particles need to be evaluated, as well as derivations of the models they invoke. It is also of note that their concern was aluminum, although their distribution was based on titanium samples, and their work is applied to other metals by other authors (Harlow & Wei, A Probability Model for the Growth of Corrosion Pits in Aluminum Alloys Induced by Constituent Particles, 1998). This probabilistic growth was deemed to be the most appropriate for the analysis in this work.

2.4 Transition from Pit to Crack

Many researchers conclude that pits transition into cracks. Fang et al. (2009) found that blunt cracks initiated around corrosion pits, which the authors stated were acting as stress concentrators. Though they didn’t directly deal with a transition model, they did state that pits were the principal sites for crack initiation (Fang, Eadie, Elboujdaini, & Chen, 2009). In general, the transition from pitting to cracking is handled by either a critical pit size model or a pitting/cracking growth competition model. In each, the pit is handled as a surface crack with growth described by pitting kinetics (Chen, Wan, Gao, Wei, & Flournoy, 1996).

In the critical pit size model, the fatigue crack nucleates when the pit is large enough for local mechanical conditions to allow for crack growth. This is most often defined in terms of the pit producing a stress intensity factor equivalent to the factor that would be produced by a crack of equivalent depth, shown in its simplest form in Equation 2 (Chen, Wan, Gao, Wei, & Flournoy, 1996). Harlow and Wei, for example, state that pit growth continues until a critical size is reached, at which time a small corrosion fatigue crack nucleates with high probability (Harlow & Wei, A Probability Model for the Growth of Corrosion Pits in Aluminum Alloys Induced by Constituent Particles, 1998). Note that assumptions made to simplify pit and crack geometry must be applied with care in this criterion, as transition is critical in determining the relative lengths of growth phases, and therefore service life.

$$(\Delta k)_{pit} = (\Delta k)_{crack} \tag{2}$$

On the other hand, fracture mechanics dictate transition in the competition model, with transition occurring according to Equation 3, when the pit growth rate is first exceeded by the growth rate of a

crack with similar geometry, often an assumed sharp crack with the same depth (Chen, Wan, Gao, Wei, & Flournoy, 1996). Again, oversimplification can be a danger, as can assumptions on which dimension of the pit is used for the initial crack geometry.

$$\left(\frac{dc}{dt}\right)_{crack} \geq \left(\frac{dc}{dt}\right)_{pit}$$

(3)

One paper gave results suggesting that both transition models can be valid. In aluminum alloys, pit size for corrosion fatigue crack nucleation was found to be dependent on loading frequency, as shown in Figure 7. In this graph, the horizontal axis is 1/f, so frequency increases from right to left. Examining the data and trends, then, it can be seen that the stress intensity at transition decreases with increasing frequency, and then seems to stabilize and become independent of frequency. This research found that critical pit size is independent of frequency for high frequency loading, but for loading below about 5 Hz, the growth competition model criteria must also be met before a crack will nucleate (Chen, Wan, Gao, Wei, & Flournoy, 1996).

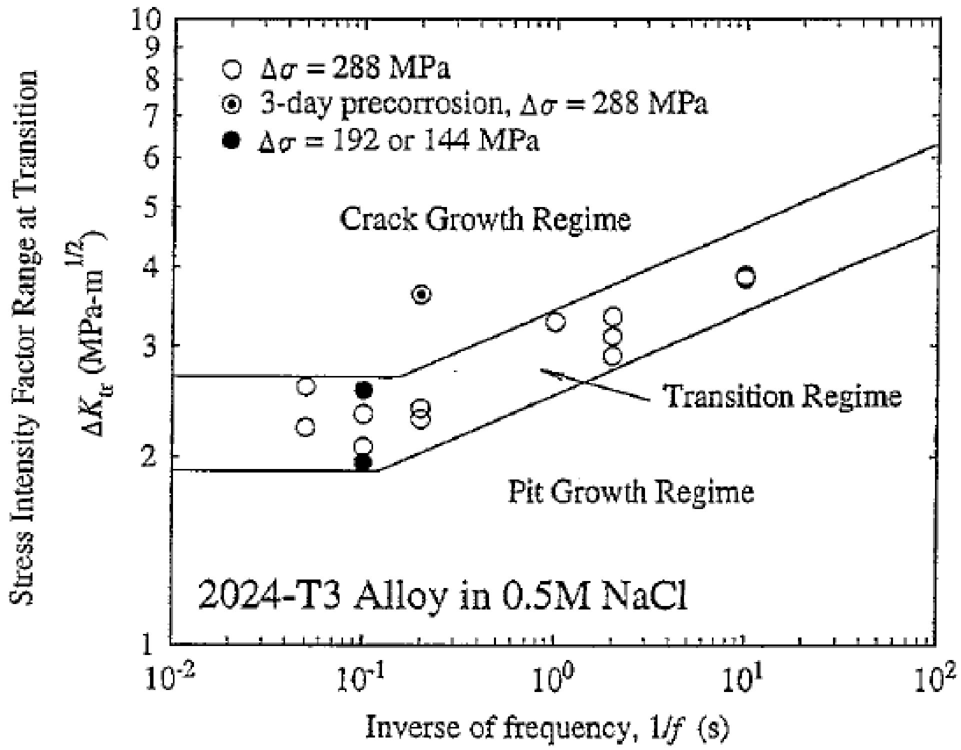

Figure 7: Stress intensity factor vs. load frequency for corrosion fatigue crack nucleation[7]

Developing this set of transition criteria further, these authors also produced Figure 8, in which increasing frequency is depicted by a line, and a series of individual frequencies. In this construct, it can be seen that for lower frequencies, pits grow for less time, transitioning quickly, due to the very high crack growth rates at these frequencies, indicated by the high slope of the f_1 line at a, for example. However, for higher frequencies, the crack growth would be lower, and the pit growth rate would dominate for a longer period, meaning that until the pit had grown sufficiently large, and its growth slowed considerably, that the crack would not form. For this reason, at high frequencies, transition would be dominated only by the necessity for a sufficiently large pit. At point a, the rapid crack growth would dominate, and the overall growth rate would increase when a crack formed, whereas for b and c, the crack and pit growth rates are equal at transition.

[7] This figure is excerpted from Chen et al., “Transition from pitting to fatigue crack growth – modeling of corrosion fatigue crack nucleation in a 20204-T3 aluminum alloy, pp 130.

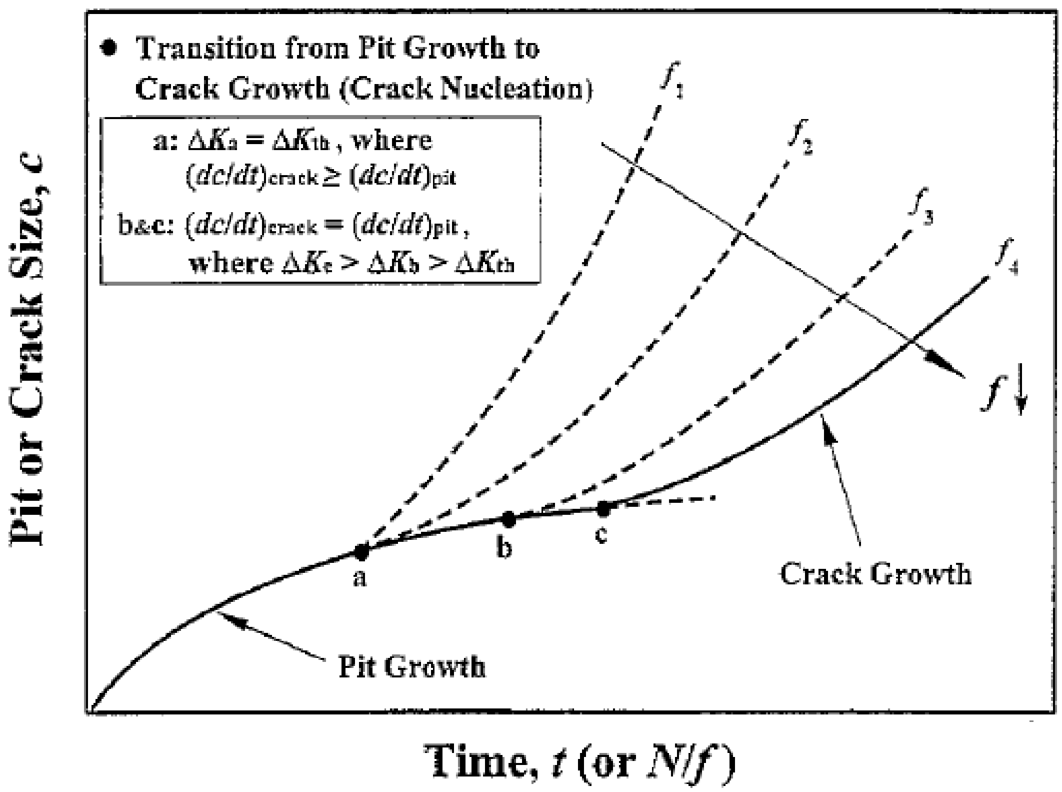

Figure 8: Conceptual framework for the damaging process of corrosion fatigue[8]

In this thesis's analysis, it is known that submarine shafts are cyclically loaded at many different frequencies, and almost exclusively below the 5 Hz transition point indicated in Figure 7. The work presented by Chen et al. (1996) was for aluminum, so it is possible that steel could transition at a different frequency, or potentially not at all. Typical submarine operations would have the shaft rotating considerably slower than 5 Hz, so it was reasonable to consider the growth rate criteria for submarine shafting. Loading is highly variable in the three regions of interest illustrated in Figure 1, with torque loading changing as the submarine changes speeds and maneuvers, and the bending frequency changing with the frequency of the shaft rotation. As loading affects crack growth rate, and therefore any transition criteria based on competition models, some consideration was given to the ramifications of assumed loading. As previously stated, investigation of cracking for this analysis is first-order only, and it was not desired for a somewhat arbitrary rate competition criterion to overshadow other factors. Each of the transition models was tested in several preliminary analysis paths, and it was found that using the simple 0.5 mm criteria from Fang et al. (2009) was quite conservative, especially with varying loading. This transition criterion is therefore applied as a mean for critical pit size.

[8] This figure is excerpted from Chen et al., "Transition from pitting to fatigue crack growth – modeling of corrosion fatigue crack nucleation in a 20204-T3 aluminum alloy, pp 131.

2.5 Crack Growth and Failure

In modeling the failure process, one pair of authors advise that a model must recognize the multiple stages of fatigue damage accumulation such as crack nucleation and long crack growth. The authors further declare that each stage is driven by different mechanisms and requires distinct modeling characteristics, as well as quantitative links that match the progression of defects from one stage successively onward (Tryon & Cruse, Probabilistic Mesomechanical Fatigue Crack Nucleation Model, 1997). Up to this point in the chapter, the current analysis has evaluated the treatment of the processes that lead to cracks. Several sources agree that these processes may account for the larger portion of the service life of components. In one example, Fang et al. (2009) found that the main fraction of pipeline life is consumed in the crack initiation process. Other research disagreed; as mentioned, the model of Shi and Mahadevan had a long short crack growth phase and a much shorter long crack phase, and their results suggest that if small pits transition into cracks, the time for short crack growth is much longer than the time for pit nucleation and growth, as was illustrated in Figure 4 (Shi & Mahadevan, 2001). For Fang et al. (2009), growth of cracks in three stages was observed. Early, blunt transition cracks became sharp cracks which grew based on proposed hydrogen interaction. The growth of these two stages consumed a majority of pipeline life compared to the long crack growth in stage three that threatened rupture and pipeline leakage, implying that crack precursors took the most time, followed by the combination of the first two phases of crack growth, and finally the terminating phase of crack growth was shortest (Fang, Eadie, Elboujdaini, & Chen, 2009).

In 1997, two researchers argued that most fatigue crack models are based on macrostructural variables, without accounting for the microstructural inhomogeneity that governs small crack growth. They stated that, in such models, propagation of cracks was reduced to the use of parametric functions of macro-level stress and strain. The challenge precluding better work was that too many micromechanical processes operated simultaneously and randomly (Tryon & Cruse, Probabilistic Mesomechanical Fatigue Crack Nucleation Model, 1997). Elaborating on this complexity, other researches declared that many models describe crack growth as a function of stress-intensity factor. This factor is a complicated function of loading, boundary conditions, crack position, and geometry, such that except for very simple or idealized geometries, significant computational difficulties arise, requiring numerical techniques including finite-element analysis that become computationally expensive (Coppe, Pais, Haftka, & Kim, 2012). Considering this challenge, and the debate in the literature about whether most of service life is spent on crack precursors and initiation or in crack growth, the current analysis evaluated the information available regarding cracking in the shaft inspection data, and found that very little data was available to select or calibrate a cracking model. Discussion with the submarine community did reveal that they traditionally use very conservative crack modeling-meaning that they design under the assumption that cracks quickly grow and lead to failure, not allotting large portions of service life to be spent in the crack growth phase.

For these reasons, a simplified crack growth model that is used by the military to predict service life was adopted. Since this thesis calculates an initial size for cracks at transition, the initial size and critical size are easy to define, and the work of Coppe et al. is readily applied. This model, based on the very popular Paris's Law, has been successful and provides information on uncertainty. The model simplifies the localized stresses to create first order approximations for remaining service life. The researchers have been able to continue to monitor the same components, updating their model and using Bayesian inference techniques to refine the model and evaluate successive iterations of predictions (Coppe, Pais, Haftka, & Kim, 2012). This has produced an effective, well-understood model that will be used in this paper for first-order approximations. It is recommended that future work updates the model in the same manner.

2.6 Uncertainty

It is widely understood that modeling often deviates from reality, in sometimes substantial ways. These deviations may result from simplifications or approximations that are well understood, but which bound the problem or make it somehow more tractable. In some cases, though, and of particular concern in this thesis, is that there can be uncertainty in whether or not the model accurately reflects and predicts the real processes and effects. A few examples of the first kind of uncertainty are provided by Melchers, in his work to consolidate data and bound the variability in it. He states that uncertainty may be due to differences in physical and chemical environments; differences between nominally similar exposures; differences between specimens under nominally identical exposures; and errors in data observation and recording (Melchers, Probabilistic Model for Marine Corrosion of Steel for Structural Reliability Assessment, 2003). Another group, addressing the scatter in empirical models, identifies that sources of scatter can also be unknown or unaddressed, i.e. the second type of uncertainty, and must be attributed to things like incomplete data and missing model parameters. They go on to point out that that the large scatter seen in fatigue testing demands that many specimens must be tested in order to establish confidence (Tryon & Cruse, A Reliability-Based Model to Predict Scatter in Fatigue Crack Nucleation Life, 1998). In general, uncertainty from scatter must be tracked and managed, and its effect on predictions taken into account in order to provide estimates and ranges of confidence in the reported results. This form of uncertainty has, as will be seen, a profound effect on predictions and the level of work that must be accomplished in order to establish restrictions on water ingress that are adequate to confidently approve a 12-year shaft service life. In other cases, though, there are types of uncertainty that are unknown or unmeasurable and that may invalidate the model and its predictions. One has already been mentioned in the missing physics discussion of King et al. Continued experience, confirmation and Bayesian updating of the model with increasing amounts of inspection data, and additional data that confirm the assumptions and decisions made in producing the model are the correct protections for these uncertainties. The current analysis must then address uncertainty in each of its modeling phases.

On one hand, Melchers contends that the models (especially corrosion models) currently available are largely empirical with wide uncertainty, which requires caution in their use (Melchers, Probabilistic Model for Marine Corrosion of Steel for Structural Reliability Assessment, 2003), yet on the other hand even he points out that relatively small differences in the composition of the steel and its heat treatments theoretically should have little bearing on its corrosion properties under conditions similar to those experienced by submarine shafts, demonstrated both in short-term laboratory experiments and numerous long-term field observations (Melchers, Probabilistic Models for Corrosion in Structural Reliability Assessment - Part 2: Models Based on Mechanics, 2003). For example, for aerobic and early anaerobic conditions, Melcher's found the coefficient of variation was between 3% and 7% for a range of coupons in waters believed to be a reasonable approximation for at-sea conditions for variability testing (Melchers, Probabilistic Model for Marine Corrosion of Steel for Structural Reliability Assessment, 2003), indicative of less scatter from this particular part of the modeling problem. Coefficient of variation, or COV, is a method for reporting variation, in which instead of standard deviation being reported directly, it is reported as a percentage of the associated mean, e.g. with a mean of 50, reporting a standard deviation of 10 or a COV of 20% would be equivalent. The analysis in this thesis therefore uses published information on rates and the uncertainty associated with those rates, instead of detailed immersion or coupon test results from which the published rates have been derived.

Pit size distributions have been studied in depth, and many distributions have been shown to predict pit sizes accurately. Harlow and Wei cite previous work from Engelhardt and Turnbull on a method shown to predict depth of pits down the major axis of ellipsoidal pits with < 10% error (Harlow & Wei, A Probability Model for the Growth of Corrosion Pits in Aluminum Alloys Induced by Constituent Particles, 1998). As the method is independently available in Harlow and Wei's paper, a discussion of the work of Engelhardt and Turnbull is not repeated in this paper.

Fatigue data, however, was found to have significant scatter. The coefficient of variation (COV) of fatigue life testing ranged widely, depending on material and loading. Even well-controlled laboratory testing saw a range of COV from less than 10 percent to over 500 percent for different steels (Tryon & Cruse, Probabilistic Mesomechanical Fatigue Crack Nucleation Model, 1997). In the analysis presented here information was taken from (dated) fatigue tests performed by the submarine community. In one paper, the authors note that a thorough investigation of the scatter in fatigue life had not been performed for most alloys. One notable exception, the work of Bastenaire, demonstrated that scatter increased with strength. The COV for mild steels ranged from 20% for low cycle fatigue to 50% for high cycle fatigue. Higher strength materials exhibited much larger scatter in fatigue life, with corresponding COV values of 25% and 90% (Tryon & Cruse, A Reliability-Based Model to Predict Scatter in Fatigue Crack Nucleation Life, 1998). Due to the large scatter in fatigue data, the parameters of the crack growth method were set conservatively, and tested during sensitivity analysis.

The analysis in this thesis acknowledges that high uncertainty can lead to very large amounts of conservatism and thus necessarily increased margins, adversely affecting cost, schedule, and performance (King, Arsenlis, Tong, & Oberkampf, 2012). Unfortunately, in the present case a paucity of

validation data leaves few options except to include in the modeling all of the sources of uncertainty discussed above and to continue to look for sources of information or other methods to reduce the uncertainty. Suggestions for further work in this area include recommended inspections and tests to reduce the uncertainty, thereby improving the accuracy and reliability of predictions made.

3.0 Research Methods

A number of models and probability distributions are being combined for the analysis in this thesis. As stated, this thesis follows the work of Shi and Mahadevan, using a log normal distribution for pit nucleation. The mean was set to 1500 days and COVs of 5%, 50%, and 95% were used. A summary of all random variable inputs is shown in Figure 10. Initial pit size modeling is also according to Shi and Mahadevan, using a normal distribution with a mean of 1.98 micrometers and the same set of COVs. To calculate initial pitting current, k is taken as a Pareto distribution in accordance with Harlow and Wei. Several values were used for each of the scale and shape parameters, in order to perform sensitivity analysis. The values of 4, 8, 10, 20 made up the set for scale parameters and 1, 3, and 5 were used as shape parameters. Detailed analysis of these distributions showed that the combination of shape equal to 1 and a scale of 4 gave results most similar to the data they reported (Harlow & Wei, A Probability Model for the Growth of Corrosion Pits in Aluminum Alloys Induced by Constituent Particles, 1998). Critical pit transition size was taken as Fang et al.'s 0.5 mm, but modeled as a normal distribution with COVs of 5%, 50%, and 95% for sensitivity analyses. This criterion gives an initial crack size, assumed to be of same length as the width of the pit, and the crack growth model of Coppe et al. is applied to calculate remaining lifetime. Hemispherical pits are assumed, which is reasonable given the expected low aspect ratio of corrosion pits, as opposed to the high aspect ratio of pitting corrosion as previously discussed. The pit growth model, covered previously as Equation 1, is:

$$\frac{2}{3}\pi\emptyset_k^2(a^3 - a_0^3) = \frac{MI_{p_0}(k)}{nF\rho} exp\left[-\frac{\Delta H}{RT}\right] t$$

The time to grow a critical pit, i.e. a pit that transitions into a crack, is:

$$t_{pg} = \frac{2\pi nF\rho}{3MI_{P_0}(k)}\left(a_{ci}^3 - a_0^3\right)e^{\Delta H/_{RT}} \tag{4}$$

where t_{pg} is the time for critical pit growth, a_{ci} is the critical pit size, and the other variables are as previously defined for Equation 1. Once transition had occurred, growth of the crack was modeled using Coppe et al.'s version of Paris's Law:

$$\frac{da}{dN} = C(\Delta K)^m. \tag{5}$$

Here, a is the crack size, N is the number of cycles, and ΔK is a range of the stress intensity factor. C and m are crack growth parameters, in this analysis established by estimating the average time between

transition and inspection, and setting these parameters to grow visible cracks approximately the size of those reported (anecdotally) in inspections.

Water ingress was modeled using a two parameter Weibull distribution. Using these distributions, the modeled version of the failure chain from Figure 2 is illustrated in Figure 9. Here, the distributions are listed for each stage; corrosion is included as part of the pit nucleation, and failure is defined as a crack that grows until it spans the circumference of the shaft.

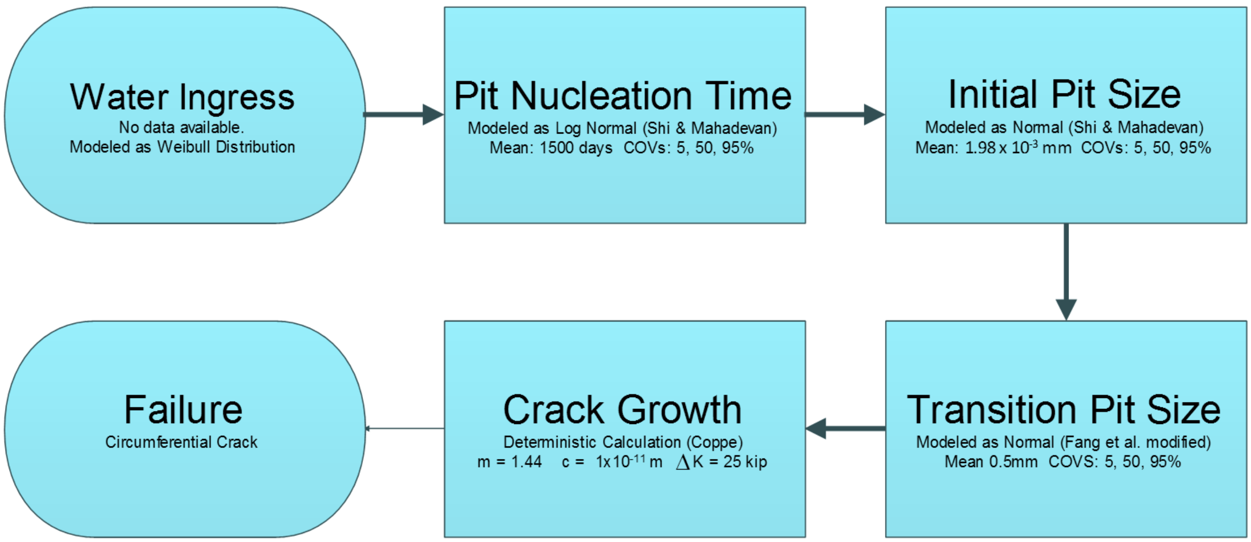

Figure 9: Summary of the failure chain as modeled

With these models in place, a Monte Carlo simulator was constructed to select all random variables and calculate the times and events of interest. A summary of the random variables and distributions is given in Figure 10.

Parameter	Symbol	Distribution	Source	Values	COVs tested (%)
Time to pit initiation		Log Normal	Shi & Mahadevan	1500 days	5, 50, 95
Initial pit size	a_0	Normal	Shi & Mahadevan	1.98×10^{-3} mm	5, 50, 95
Critical pit size	a_{ci}	Normal	Fang et al. (modified)	0.5 mm	5, 50, 95
Cluster size	k	Pareto	Harlow & Wei	Scale: 4, 8, 10, 20	Shape: 1, 3, 5

Figure 10: List of probabilistic distributions and parameters in use

Other parameters utilized in the modeling are listed in Figure 11.

Loading (each side of cycle)	25,000 psi
Cycles per month	1,000,000
Valence	2
Faraday's Constant	96,485 J/volt-gram equivalent
Density of steel	8000 kg/m^3
Molecular weight (Iron assumed)	55.845 g/mol

Figure 11: List of parameters for modeling

Once established, the values listed were held constant during investigation of potential water ingress distributions. Values were changed for sensitivity analysis later. The scale and shape parameters for the Weibull water ingress distribution were manipulated as the independent variables.

Using these distributions and parameters, the simulator calculated times to water ingress, then additional time to pit initiation, followed by time to growth until transition into a crack, and finally time for cracking to lead to failure.

Outputs of the simulator were the percentages of simulated shafts that had wetted shafts, pitted shafts, cracked shafts, and failed shafts after 6 simulated years of exposure and operation. Based on the actual shaft inspection results, target values were:

Condition	Inspection Summary Value
Wetted	70%
Pitted	40%
Cracked	4%
Failed	0

Figure 12: Summary statistics, used as target values

For each potential water ingress distribution, a distance metric was calculated from the target values. The deviations between the output values and the target values were taken, squared, and summed. The distance metric was the square root of this sum, i.e. the definition of the L2 norm with 4 variables. No weighting was applied, with the exception that water ingress distributions that consistently gave failures were rejected. This distance metric was minimized by successive changes to the two Weibull parameters and tracking of statistics on the norm and standard deviation of the metric.

Next, a similar procedure was followed for investigating the allowable level of water ingress to produce acceptable 12-year inspection results. It was determined that wetting and pitting are much more indicative of the states of processes than desirable metrics of performance, when compared to cracking and failures. That is, the submarine community is arguably much more concerned about having a very low probability of a failed shaft than it is about whether a given percentage of those shafts are wetted. For this reason, the target values for the 12-year simulations changed. Water ingress distributions were manipulated in much the same way as before, with the new goal of producing zero failures on a consistent basis, while allowing for some small percentage of cracks to develop. Pitting and wetting were tracked for reporting purposes, but these values were allowed to deviate as necessary from any target values.

For each distribution and parameter selected, the uncertainty is handled in one of several ways. When distributions have been directly selected, the parameters have been chosen to be consistent with information available on uncertainty in the data that drove the selection of the distribution. In several cases, a set of COVs has been utilized to broadly capture some relatively unknown scatter in the data, with the varying levels of the COV intended to help quantify the effect of the parameter and its scatter

on the overall model and predictions. Finally, in a few cases a parameter was simply set to several values, and the effect of the changes was tracked on the results of the simulation. In all cases, the range of results have been reported back to the submarine community for their consideration, though direction for the project has mostly provided focus, and dictates the results shared in the current paper.

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4.0 Results and Discussion

4.1 6-Year Allowable Wetting Distribution

The minimum L2 metric for allowable water ingress to achieve the 6-year inspection result was given by a Weibull distribution with shape parameter 0.75 and scale parameter 1600. The probability density function (pdf) is shown in Figure 13. The high level of positive skewness is very evident in this image. This would be indicative of shafts having a relatively high probability of getting wet early in life, here peaking around the end of the first year, near the 300 day point.

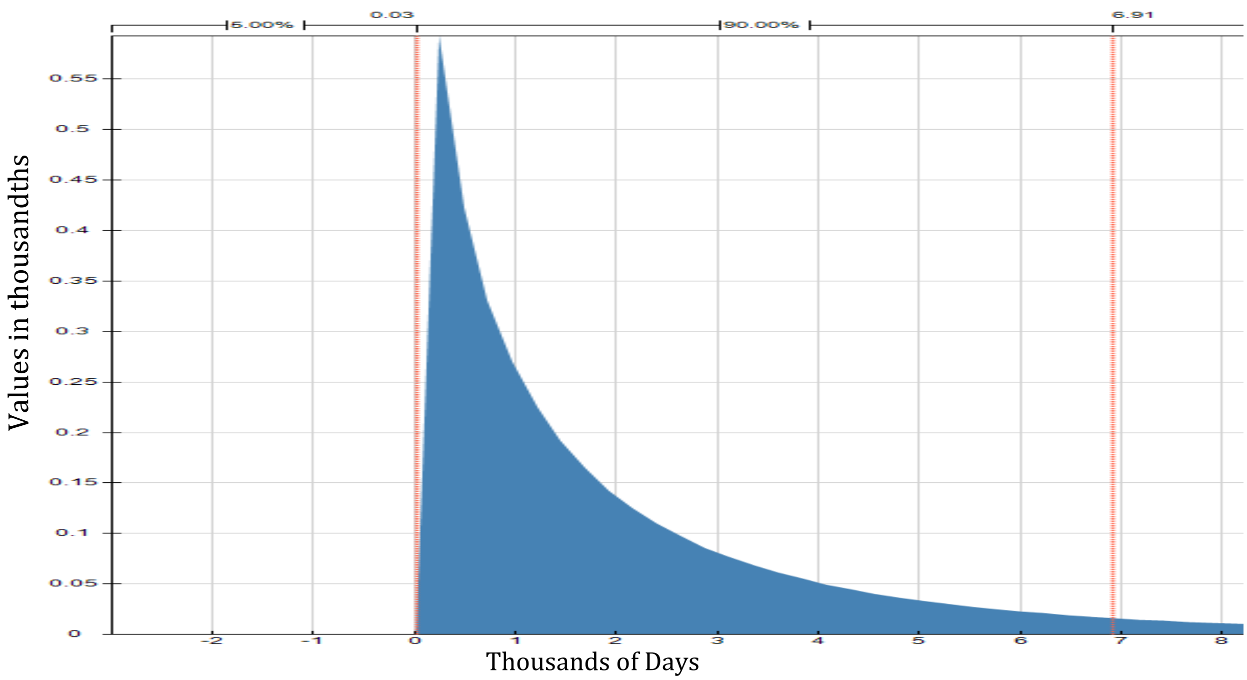

Figure 13: PDF of 6-year allowable wetting showing high skew

The cumulative density function (cdf), shown in Figure 14, illustrates the total number of shafts wetted as a function of time. The vertical lines indicate the 6 and 12 year points for reference (2190 and 4380 days). Note this distribution accurately predicts a value of approximately 70% wetted shafts at 6 years, consistent with the target value.

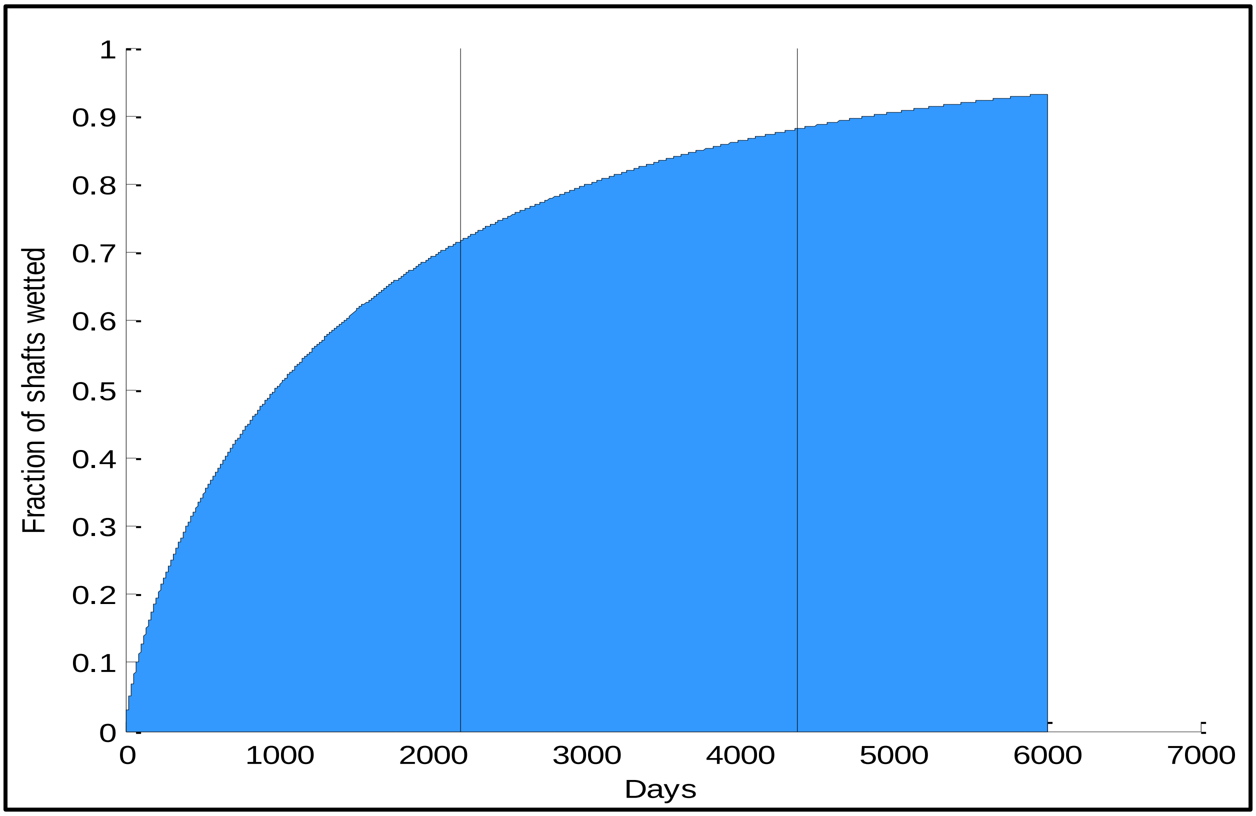

Figure 14: CDF of 6-year allowable wetting

The high probability of wetting early in operational life must be correlated to physics that are reasonable. Several cases are plausible. First, it is possible that on some shafts, the glass-reinforced plastic cover wears in rapidly, and experiences separation from the underlying metal and/or painted coatings early in life, allowing water to wick down the separation and onto the metal shaft. A second possibility is that the GRP or an O-ring fails almost immediately upon entry to service, and the water then takes some time to migrate through the paint coating, or the motion of the shaft cracks the paint over time, to allow the water to reach the bare metal. In this scenario, it is also possible that the true distribution of water ingress is bimodal, with some percentage of very fast or instantaneous failures, and the remaining shafts being provided much more dry operational time by the protective systems. Investigation into this possible bimodal distribution is recommended for future efforts.

While the specific number of actual inspections upon which the inspection summary values are based was not provided, there have been between 50 and 100 inspections completed. Through successive

runs of the Monte Carlo model, the prediction was that in a sample size of 60, there was actually about a 10% chance of having experienced a single shaft failure. This result became the basis for defining "similar performance" for a 12-year inspection interval.

4.2 12-Year Allowable Wetting Distribution and Comparisons

The minimum L2 metric for allowable water ingress to achieve these 12-year inspection results was given by a Weibull distribution with shape parameter 2.14 and scale parameter 32,000. The pdf for this distribution is shown in Figure 15. The scale of this distribution must be noted; for the 6-year distribution in Figure 13, a period of 8,000 days covered the distribution, excepting the final tail, while the 12-year distribution can only be illustrated on a scale closer to 80,000 days. The skew is also substantially less, meaning that not only must the dry time increase significantly for shafts, but there is much less allowance for some shafts to get wet very early in life. Whatever the physics driving the very early failures is proven to be, that problem must be identified and mitigated.

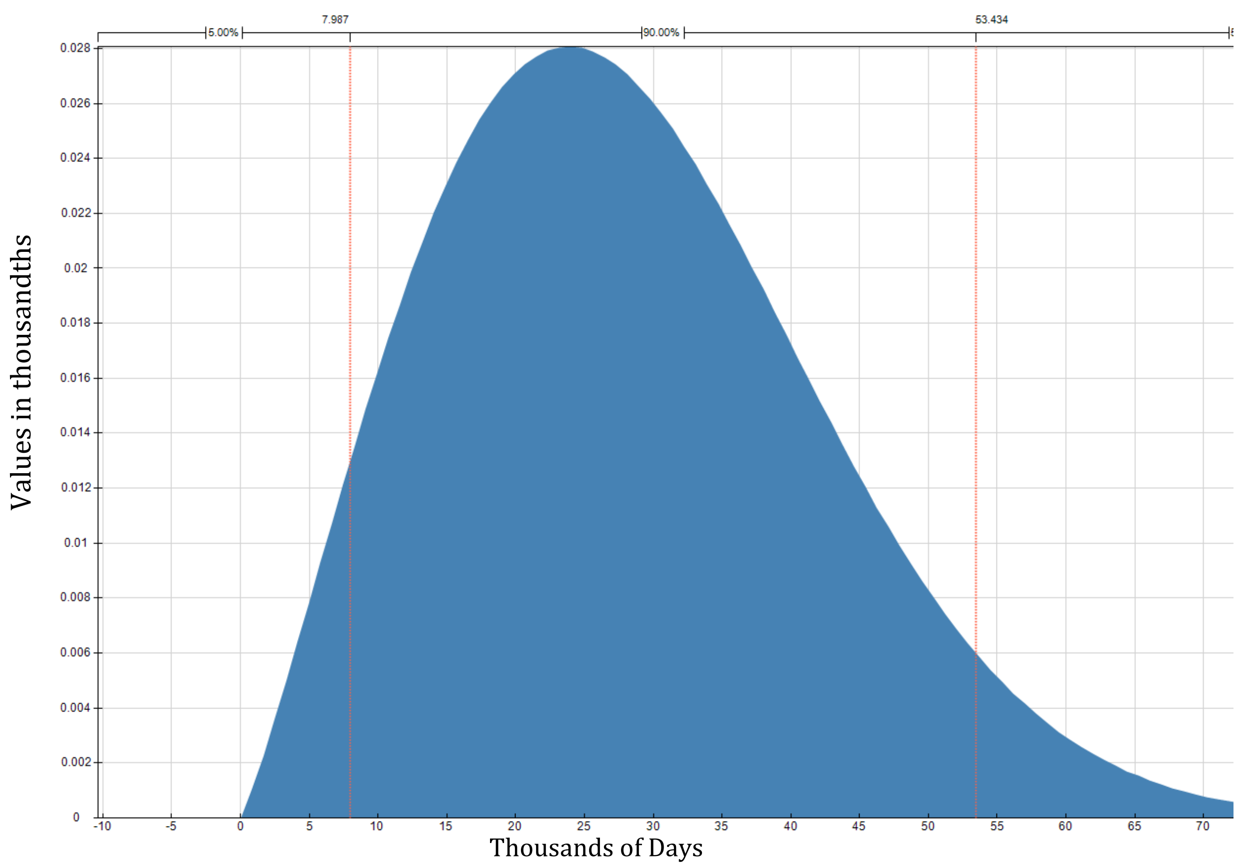

Figure 15: PDF of 12-year allowable wetting

The associated cdf in Figure 16 again shows the difference in scale between the two requirements for water ingress prevention. The vertical lines still shows the 6-(and 12-) year points for reference, but the 6-year point now correlates with a small percentage of shafts being wetted by this time, instead of nearly 70%. The stark contrast in the two distributions is better illustrated in Figure 17, where the two distributions are illustrated together, with the 6-year in blue and the 12-year in red. The magnitude of the change is evident, and illustrates a significant challenge to the design of the improved water ingress prevention.

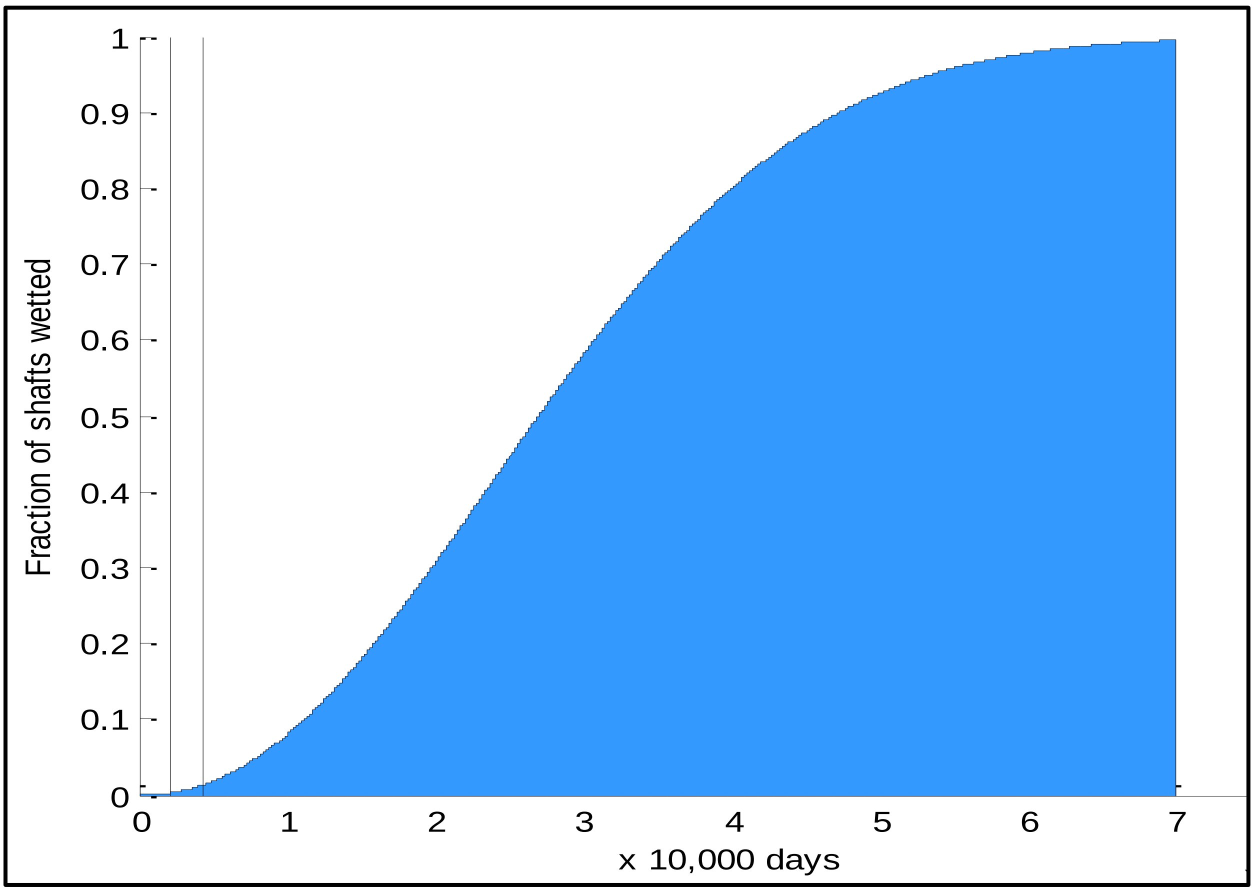

Figure 16: CDF of 12-year allowable wetting

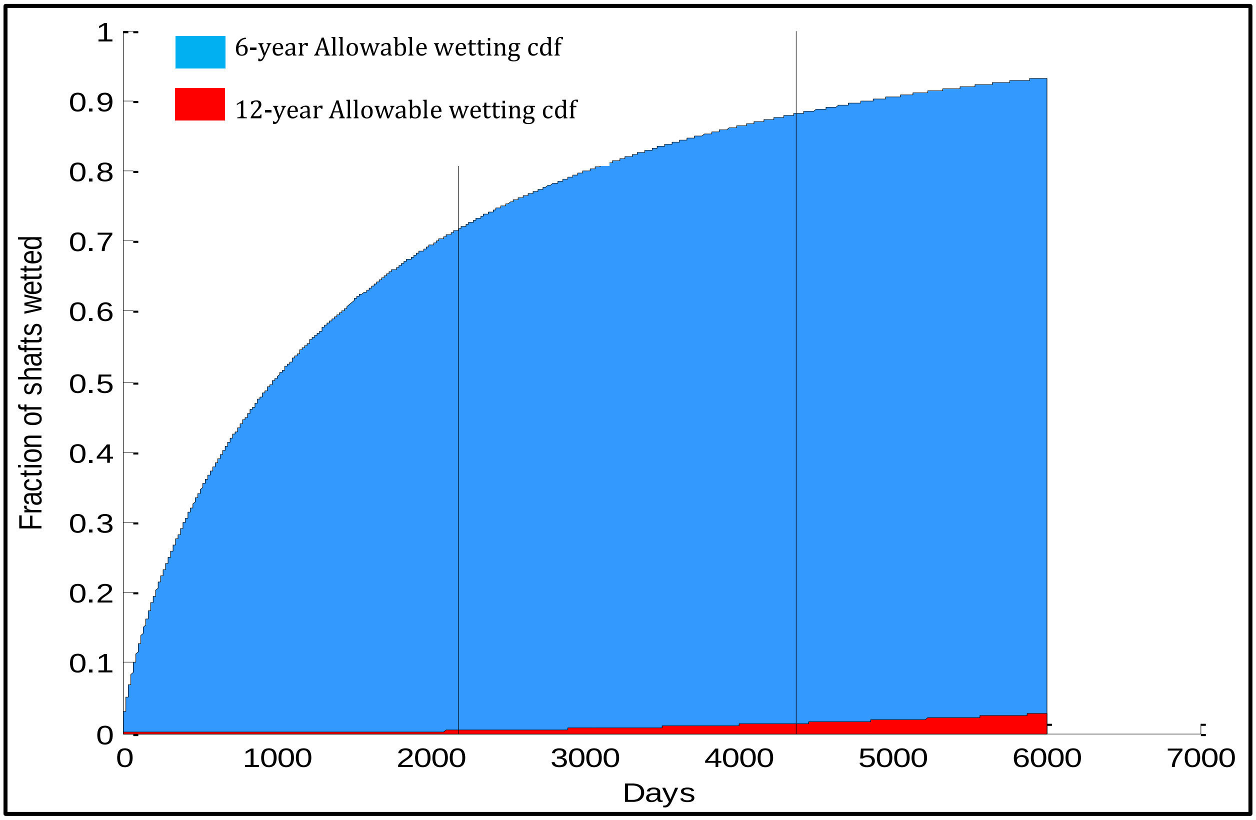

Figure 17: CDFs for 6-year (blue) and 12-year (red) allowable wetting

More detailed comparisons of the allowable wetting for satisfactory 6 and 12 year inspection intervals are provided in the tables in Figure 18 and Figure 19. Each table provides the predicted inspection results for shafts for each of the water ingress distributions covered. Figure 18 gives the inspection results if shafts are inspected at the 6 year point. Note that for the 6 year allowable

Evaluated at 6 Years	Allowable Wetting Profile for 6 Year Service Life	Allowable Wetting Profile for 12 Year Service Life
Wetted	71%	2.3%
Pitted	39%	0.04%
Cracked	5%	0
Failed	0*	0

Figure 18: Prediction of inspection results at 6 years for each water ingress distribution

ingress distribution, with 6 years between inspections, there is an estimated 10% chance of breaking a shaft before the inspection and refurbishment interval is reached. The same is true for the 12-year allowable water ingress distribution when inspections are set to 12-year intervals.

Evaluated at 12 Years	Allowable Wetting Profile for 6 Year Service Life	Allowable Wetting Profile for 12 Year Service Life
Wetted	87%	5.6%
Pitted	69%	0.3%
Cracked	59%	0.16%
Failed	45%	0*

Figure 19: Prediction of inspection results at 12 years for each water ingress distribution

Examination of these tables indicates that keeping shafts dry greatly reduces the risk from corrosion fatigue, and the percentages listed are consistent with the order of magnitude increase in dry time required, according to this analysis. If water ingress is prevented according to the requirements for 12-year inspection intervals, but shafts are allowed to stay in service until failure, the predicted failure distribution is shown in Figure 20. Total years in service until failure is noted on the x-axis, and the vertical axis on the left lists the number of shafts failing in a corresponding 10-year period (e.g. from 111-120 years, at the 120 datum). The shape of this graph is dominated by the very long times before wetting can be tolerated, seen by the similarity in shape and scale between Figure 15 and Figure 20.

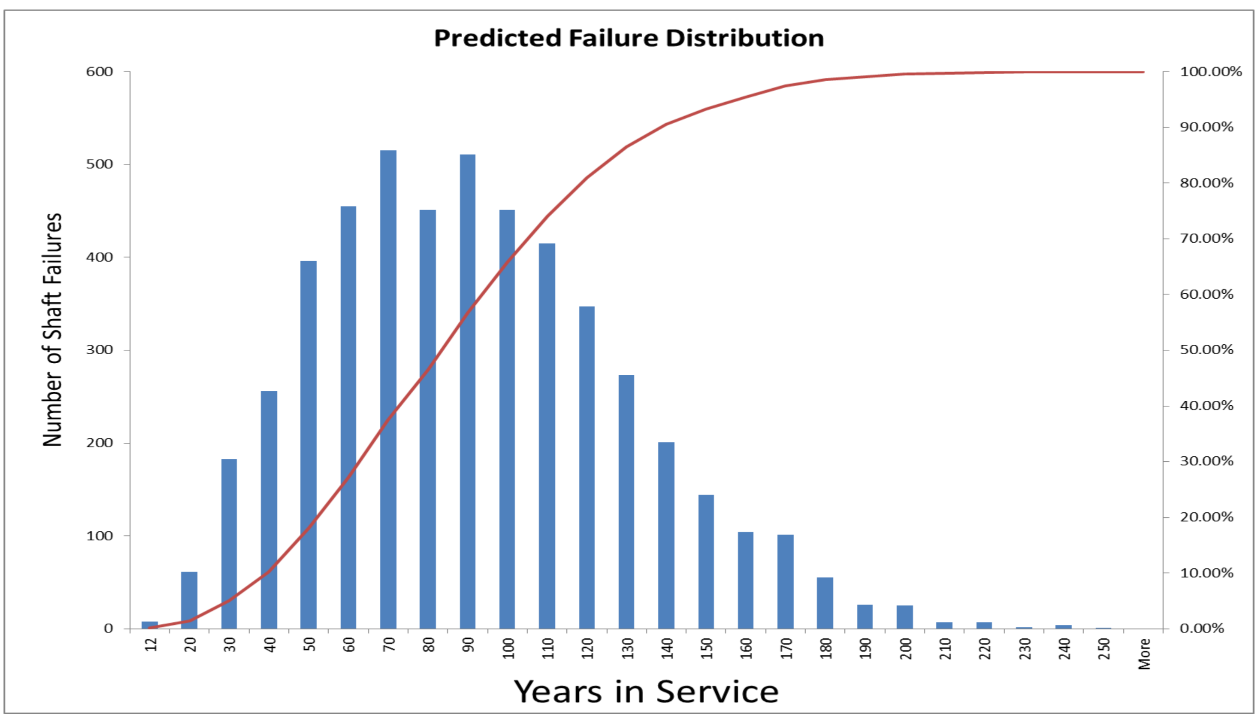

Figure 20: Predicted failure distribution (one representative simulation)

The cumulative frequency of predicted failures, a close approximation to the cdf of this distribution, is overlaid on the chart in Figure 20 in the red line, with the percentage scale on the right of the figure. These two depictions each indicate that, at the 12-year inspection point, there is a low probability of failure. Though the distribution of failures is defined by samples and frequencies, the shape of the curve suggests that it, too, could be approximated by a Weibull distribution, which is not uncommon for failure analyses. Additional detail is provided in Figure 21, which expands the information in the early years of the histogram presented in Figure 20. Two year bins are used for the first 20 years, detailing the times of failure. In this individual simulation of 5,000 shafts, it can be seen that 3 were predicted to fail before the 10 year point, with an additional 3 by the 14 year point. Successive simulations gave similar results, revealing a predicted failure probability of about 0.14%, in terms of shafts that fail prior to the 12 year inspection interval. It is worth noting that this probability, if accurate of the true process, is unacceptably high; typical Navy risk management goals for a component failure of this magnitude would require 0.0001%, or closer to 1/1000th of the estimated probability. There is, however, considerable uncertainty in this estimate, and this uncertainty requires some consideration.

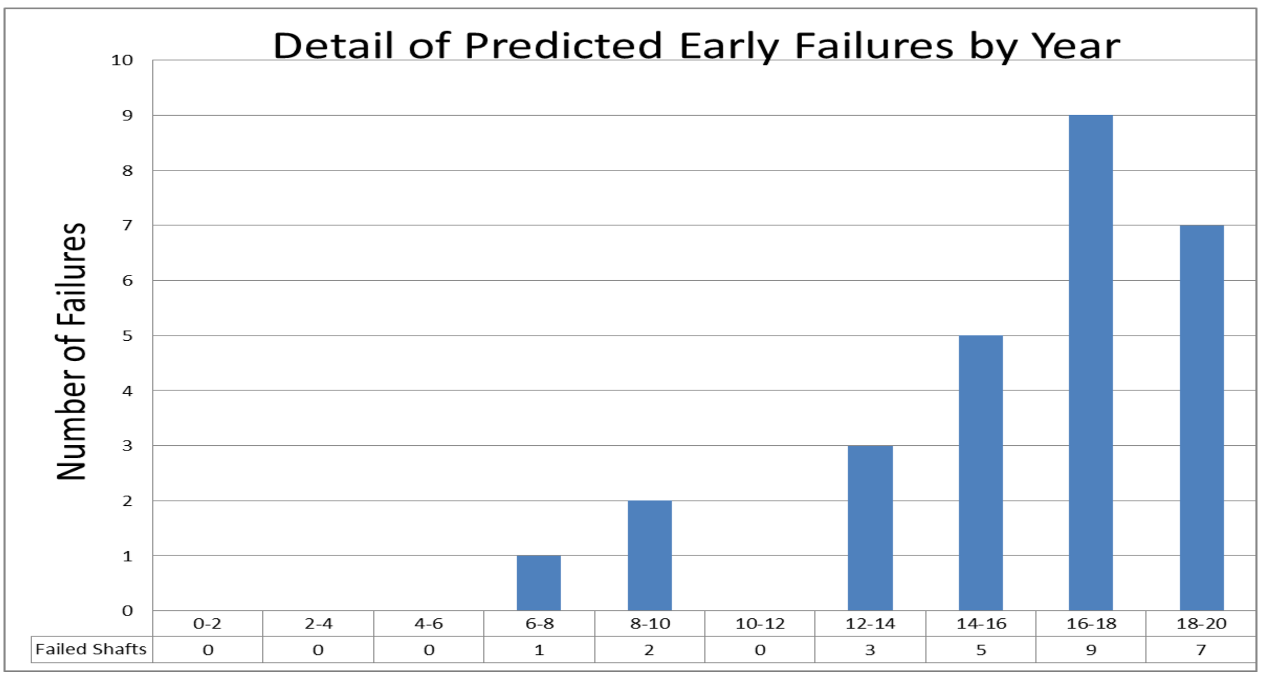

Figure 21: Detail of shafts predicted to fail early (one representative simulation)

4.3 Uncertainty

King et al. (2012) declare that predictions are most useful in the presence of quantified uncertainties. As discussed previously, this analysis includes a number of sources of uncertainty. Figure 22 illustrates the effect of this uncertainty. In this figure, the two-tailed 95% confidence interval on the mean is

illustrated with the braces. As stated in the figure, this span is nearly 180 years, due to the large uncertainty in the estimates. This broad uncertainty has the effect of pushing the mean far to the right, forcing overdesign and undue conservatism. In this case designers would be required to design a shaft system producing a mean time to failure of approximately 100 years in order to have the predicted (not quite acceptably) low probability of a failure in the 12 year operational cycle.

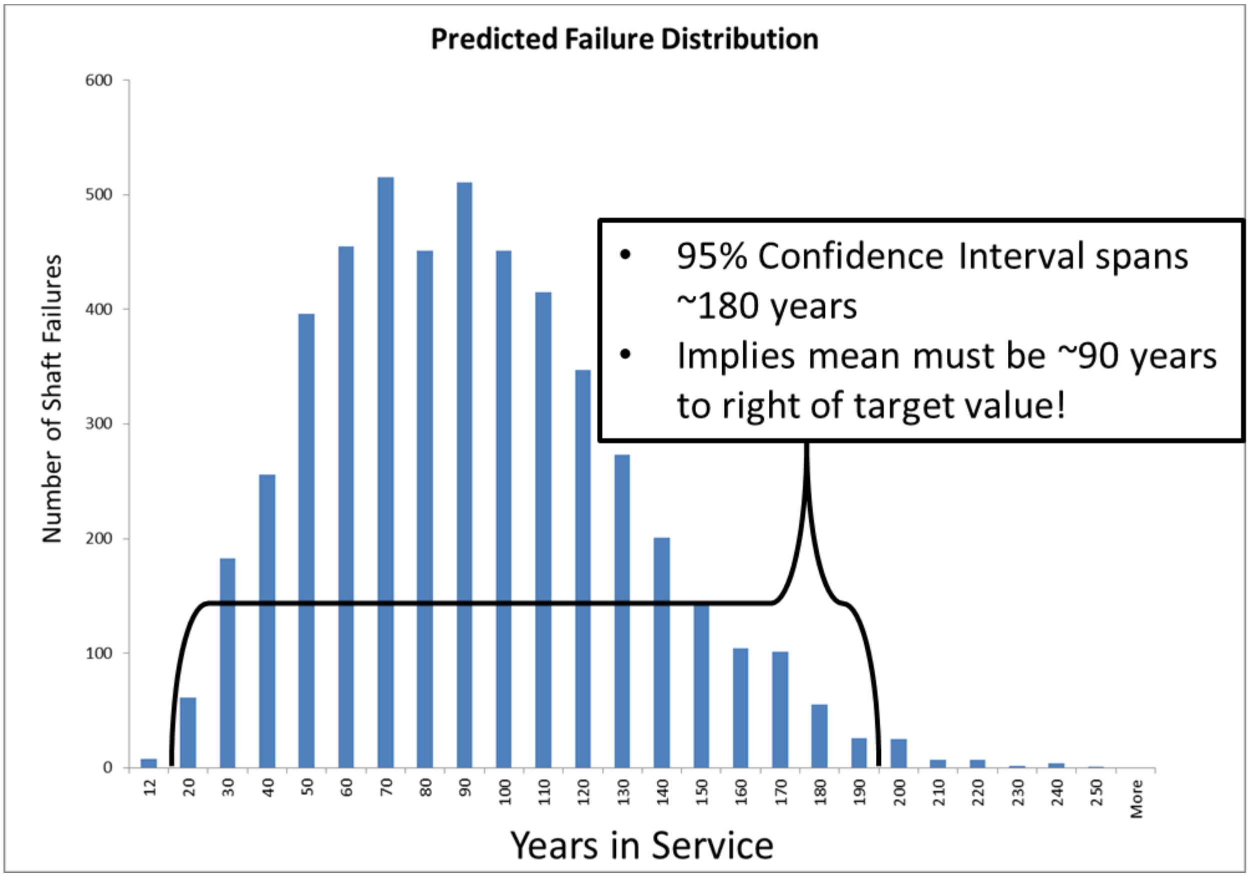

Figure 22: Sample failure distribution showing effect of high uncertainty

Reducing this uncertainty would have a number of benefits as discussed, and there are many methods to accomplish this. Improved inspection data is one of the key recommendations of this thesis, to include characterization of the types of indications and details about the distribution of their sizes, shapes, and locations. Targeted testing could also replace some of the COV dispersion estimates with specific values. Reducing uncertainty is doubly important when one considers that little data or testing exists on the effectiveness of the current and proposed systems for preventing water ingress. Claiming (or assuming) that these improvements will achieve the goal of a 12 year inspection interval is therefore tentative at best, at least for the foreseeable future while data is accumulated, and it will be difficult to build a strong case that the target probabilities have been reached. The effects of each parameter on this uncertainty, and on the estimates of shaft life, are investigated in Appendix B: Sensitivity Analysis.

5.0 Conclusions and Recommendations for Future Work

This thesis has utilized modeling methods from literature to evaluate the corrosion fatigue failure process of the submarine propulsion shaft. Using these models, this thesis was able to infer information about the unknown, precipitating water ingress distribution, and about the level of changes necessary in preventing water ingress in order to achieve a reliable 12-year inspection interval. Although preventing water ingress is a desirable method for improving shaft life, as it interrupts the failure chain at the earliest possible point, the level of uncertainty in modeling this process complicates and calls into question the level of improvement required. This thesis also made first order estimates of the failure distribution of propulsion shafts, noting that the predictions do not quite achieve typical levels of confidence for navy risk management.

In order to achieve the 12-year inspection interval, shafts must stay dry for considerably longer, defined in this thesis in the terms of an order of magnitude longer. Uncertainty drives much of this time, and reductions in uncertainty would greatly improve the reliability of these results, as well as reduce the level of water ingress prevention shown to be necessary.

Improved inspections have been emphasized as a necessary step to achieving the desired levels of confidence, both in the reliability and in the predictions being used to justify the inspection interval. A broader canvassing of the literature is further recommended, that additional methods and models might be tested, to create a more complete picture of the predictions that might be made by different methods. A recommendation based on King et al. (2012) would be to manufacture one or two additional shafts, allotting resources later in the class life to take these shafts out of service for detailed analysis, treating them as application tests. This would both reduce the difference in the period of prediction and improve the quality of data available. As mentioned earlier, an investigation of the corrosion rates of interest would benefit designers for all future designs. Natural seawater, as pointed out several times by Melchers, contains biological and chemical components that affect the corrosion behavior. Additionally, these constituents may affect the initial oxidation potential and therefore corrosion rate. Controlled experiments are recommended that use specimens exposed to natural seawater, to approved artificial seawater, and to an emerging seawater substitute that uses artificial seawater augmented by enzymes that mimic the effects of the components that so concern Melchers (2003). Another investigation recommended in an earlier section is analysis similar to that of this thesis, but using bimodal distributions simulating some failures upon entry-to-service. Testing that reveals the water ingress path or paths is also recommended. Finally, it is recommended that methods to interrupt the corrosion fatigue failure chain at other points be investigated. As mentioned, other work by this project, not addressed directly in this thesis, has investigated a cladding material that exhibits corrosion properties which may preclude pitting. This was a material of opportunity, exhibiting some of the traits desirable in a cladding for the shaft, but a more detailed analysis would need to be performed to design a material with the full suite of material, mechanical, and corrosion properties that would be desired in this application.

Two interesting results were also identified during sensitivity analysis. Due to the long time that shafts must be kept dry to achieve the desired results, above, very few of the other parameters tested had a large effect on the final failure distribution, which was primarily driven by the "safe" dry time of each shaft. However, the pitting current was seen to have a substantial effect, especially interesting given the likelihood of a galvanic couple and high corrosion current, discovered as mentioned by other work from this project, which is not the focus of this thesis. Second, the transition criterion in use had a significant effect on which predictions showed the greatest deviation from the target values, for cases with minimized L2 norms. Testing is recommended to confirm the critical pit size and transition criteria from a pit to a crack.

List of Abbreviations

CDF	cumulative density function, also sometimes cdf
COV	Coefficient of Variance
FMEA	Failure mode and effects analysis
GRP	Glass reinforced plastic
PDF	probability density function, also sometimes pdf
SRB	Sulfate reducing bacteria

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Appendix A: Discussion of Pareto Distributions

Though many will be familiar with other concepts from his work, Italian economist Vilfredo Pareto is the namesake of a less commonly known distribution function, or more accurately a family of distribution functions. One of the primary concepts typifying these distributions is that, though possible, the likelihood of high values of the object or parameter being studied becomes increasingly small. The well-known "80-20" rule actually comes from this distribution, and is often associated with the accumulation of wealth; that the upper 20% of people own 80% of the wealth. Alternatively, in process engineering and process improvement, this rule states that only a few types of problems, around 20%, make up 80% of the improvement that can be gotten-indicating that classifying and numerically analyzing the faults in a system will rapidly identify which solutions to target first.

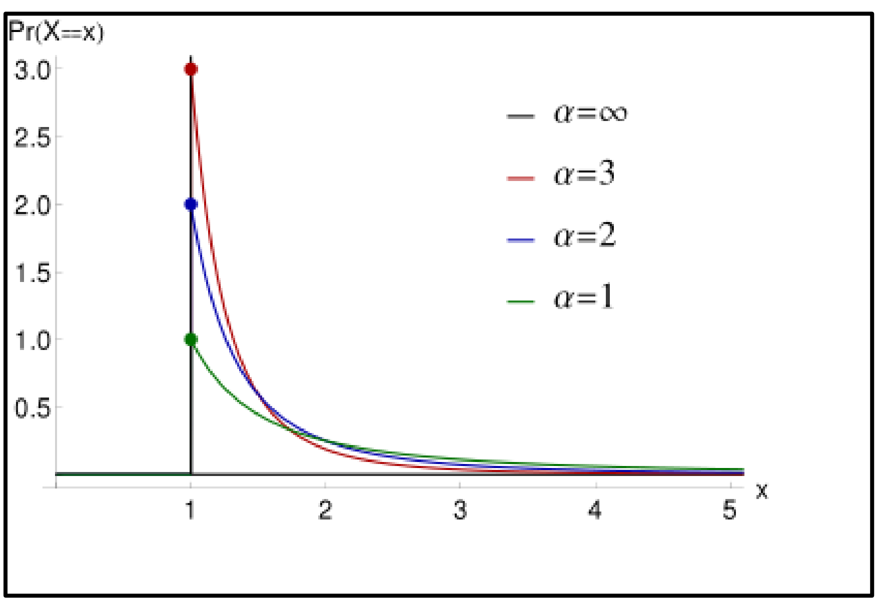
The distribution is defined as a survival function, such that it maps the probability for a random variable X with a Pareto distribution, that a given value of X is larger than some number x, and the distribution is defined including the (positive) lower bound of X, x_m. The pdf for the Pareto distribution is given by:

$$f_x(x) = \begin{cases} \alpha x_m^{\alpha} & x \geq x_m \\ 0 & x < x_m \end{cases}$$

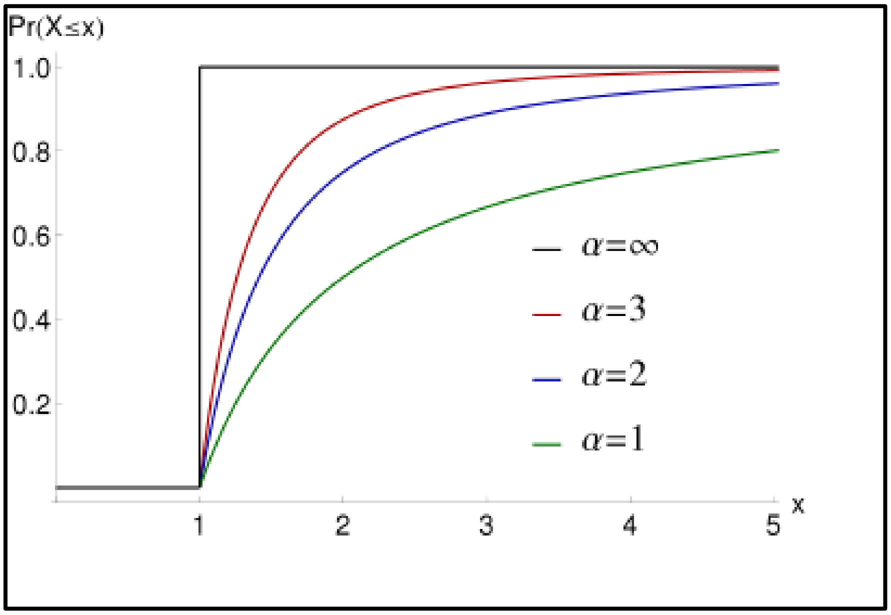
Which gives a cdf of:

$$F_x(x) = \begin{cases} 1 - \left(\frac{x_m}{x}\right)^{\propto} & x \geq x_m \\ 1 & x < x_m \end{cases}$$

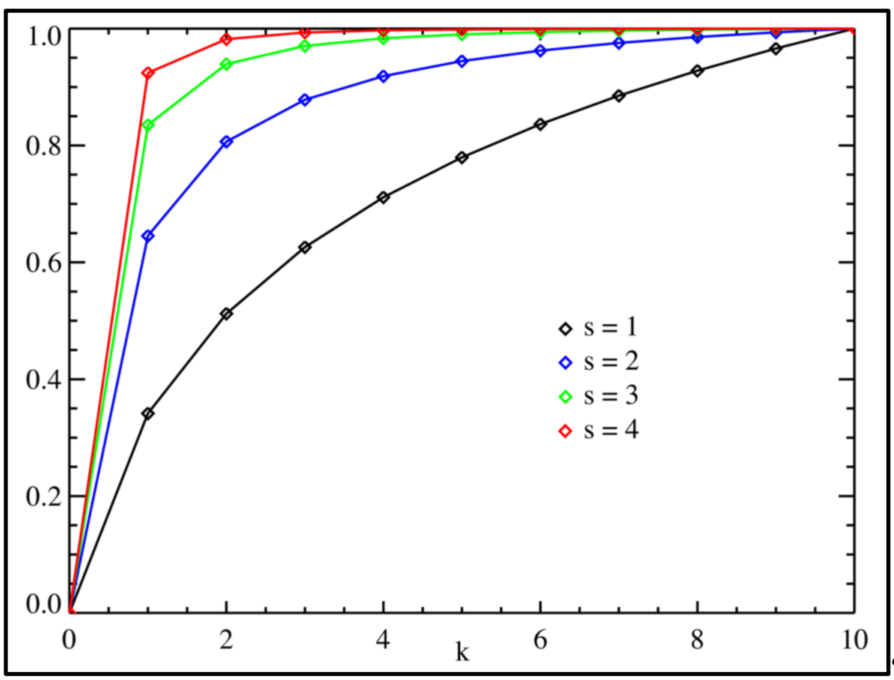
Graphically, the pdf for is a monotonically decreasing function, here shown for four values of α:



The cdf then becomes:



The discrete Pareto, used by Harlow and Wei, is more commonly referred to as Zipf's law, and its cdf is seen to be similar, approximating the continuous distribution:



[9]

[9] These graphs were taken from the Wikipedia page; a similar discussion is also available there (Wikipedia Commons, 2014)

Appendix B: Sensitivity Analysis

Parameters investigated for sensitivity analysis included the COV for the log normal pit initiation time distribution, taken from Shi and Mahadevan (2001), the COV for the normal initial pit size distribution taken from Shi and Mahadevan (2001), and the shape and scale parameters of the Pareto clustering distribution taken from Harlow and Wei (1998). The base case was the case that was coupled to the 12-year allowable wetting distribution detailed in this thesis, which had shape parameter 2.14, and scale parameter 32,000.

Parameters for the base case were:

Pit Initiation Time Mean	Pit Initiation Time COV	Pit Size Mean	Pit Size COV	Pareto Shape	Pareto Scale
1500	0.5	.00000198	0.5	1	4

After 20 runs of 5,000 iterations of the Monte Carlo each, the mean failure time for shafts was 127 years, with a standard deviation of 48.9. The effects of altering the parameters were:

		Mean Fail Time	Standard Deviation
Pit Initiation Time COV	Decreased to 0.05	Negligible	Negligible
	Increased to 0.95	Negligible	Negligible
Pit Size COV	Decreased to 0.05	0.5% increase	Negligible
	Increased to 0.95	Negligible	0.4% decrease
Pareto Scale	Increased to 8	Decreased 15% (108.7)	Decreased 8%
	Increased to 10	Decreased 18% (104.7)	Decreased 9%
	Increased to 20	Decreased 25% (96.0)	Decreased 11%
Pareto Shape	Increased to 3	Increased 12% (142.4)	Decreased 6%
	Increased to 5	Increased 16% (148.2)	Decreased 8%

These results show that the pitting current, driven by the size of the initiating cluster, is the key parameter for determining the length of time spent during pitting. This makes physical sense, as the discussion in Harlow and Wei indicates that the largest clusters do the most damage, and are most likely to transition into cracks, and all the tested increases make larger clusters more likely. Analysis showed that the base case with shape 1 and scale 4 gave results that best matched those described in their study (Harlow & Wei, A Probability Model for the Growth of Corrosion Pits in Aluminum Alloys Induced by Constituent Particles, 1998). These results indicate that more accurate and reliable predictions might be facilitated through a more in-depth study of the distribution of particles found in shaft steel.

Because the models tended to underpredict cracking, although results from the L2 norm perspective were acceptable, a second analysis was done using the distributions coupled with the 6-year allowable wetting distribution, in order to investigate further. Varying the parameters for the transition criteria revealed that using a normal distribution for the size of the pit that transitions into a crack, with mean 0.3 mm and standard COV of 0.95 made the 4% cracking criteria more accurate when coupled with a slightly different wetting distribution. When "calibrating" this criterion in this way, a new minimum L2 norm was found, with a set of distributions that predicted results even more similar to the target values of the actual inspections. The new water ingress distribution had shape parameter of 0.75, and scale increased to 1750 (from the previously reported 1600). This combination tended to slightly underpredict the number of shafts exhibiting wetting, while providing consistent results very close to the 40% pitted and 4% cracked shafts, with no failures at the 6-year point. It is recommended that a study be done to validate the transition criterion for submarine shafting.